

Paper 2 – MATERIALS

HANDOUT 2

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Section 2 covers Ex. Paper 2, Q.1-6; Section 3: Q.7-10.

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2. Elastic Properties of Materials

2.1 Elastic Stiffness in Design

Many engineering components need to behave *elastically* under load, i.e.

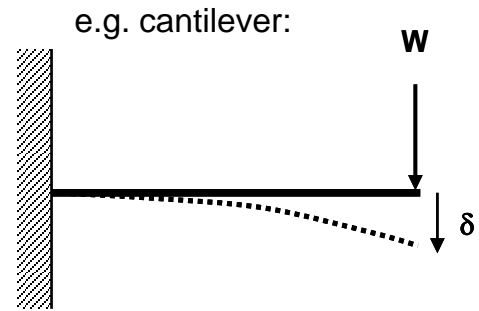
Examples:

- “structural” applications: bridges, buildings, vehicles, bicycles, furniture, machines
- “functional” applications: precision instruments, hard disk drives

The *stiffness* of a component or structure is:

Stiffness depends on:

- material (steel, wood, CFRP...)
- cross-section size + shape (tube, I-beam ...)
- mode of loading (tension, bending)



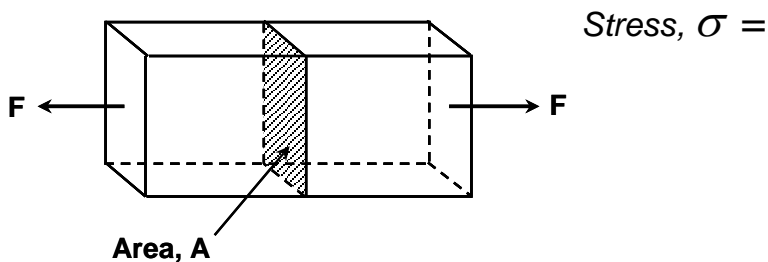
Intrinsic *material* stiffness measured by *Young's modulus, E* (defined below).

Effects of size, shape, loading: use results from IA Structures.

2.2 Stress and Strain

2.2.1 Uniaxial Tension and Compression

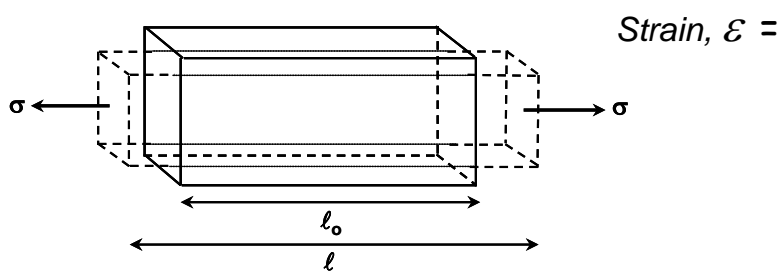
Stress



This is a *normal* or *direct stress* (i.e. force *perpendicular* to loaded area).

Stress is measured in *Pascals* (usually MPa: $1 \text{ MPa} = 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$).

Strain



This is a *normal* or *direct strain* (i.e. extension *parallel* to original length).

Strain is a ratio of lengths, so it is *dimensionless* (no units).

For most materials elastic strains are small ($< 0.1\%$) – rubber is an exception.

Poisson's ratio

In uniaxial tension:

- the material gets longer *and* thinner
- the lateral contraction and the tensile extension are proportional
- the ratio is a material property: *Poisson's ratio, ν* (Greek "nu", not "v"):

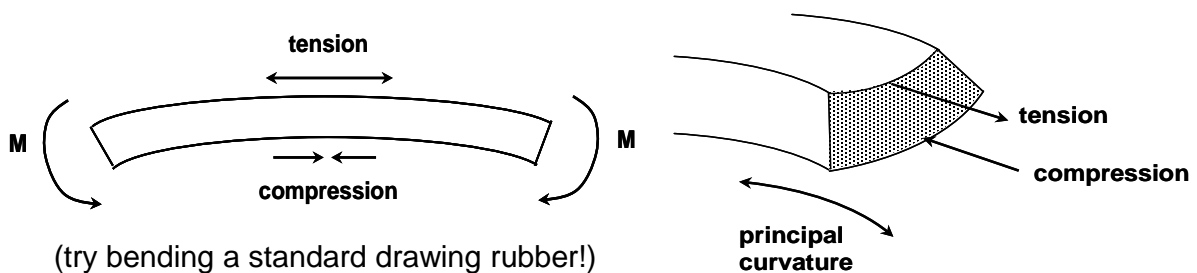
$$\nu = \frac{\text{lateral strain}}{\text{axial strain}} \quad (\text{note minus sign, so } \nu \text{ is positive})$$

Lateral strain is *not* due to volume conservation, but reflects the way atomic bonds deform under load. Typical values of Poisson's ratio, ν :

- crystalline materials (metals, ceramics)
- porous solids (foam, cork)
- elastomeric polymers (rubber)

When does Poisson's ratio matter?

- not important in most design with *uniaxial loads* (e.g. members in a truss: fractional change in area)
- important when the *stress state is 2D or 3D* (e.g. constrained expansion – examples later)
- important in *vibration of plates* (e.g. affects frequencies of vibration modes)
- important in *large strain bending*, giving *anticlastic curvature*:

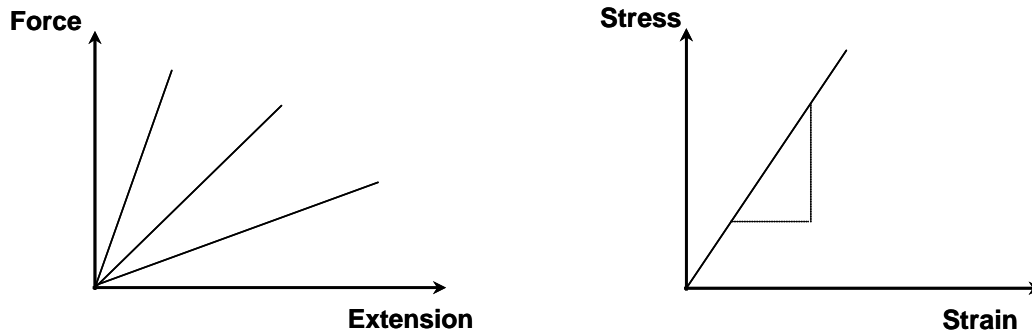


Young's Modulus

Most engineering materials are *linear elastic*: under uniaxial load, a sample of constant cross-section extends in proportion to the load (Hooke's Law).

By converting load to stress, and extension to strain:

- the response is made independent of geometry;
- the slope is then a material property, *Young's modulus, E*.



For uniaxial tension (or compression):

Young's modulus, $E =$

2.2.2 Stress and Strain in 3D

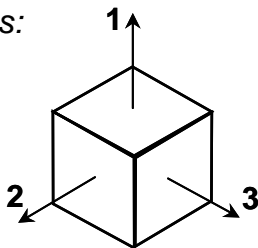
Hooke's Law in 3D

Some engineering components are loaded uniaxially, e.g. cables, struts; but often the stresses are *multiaxial*, e.g. beams, arches, pressure vessels.

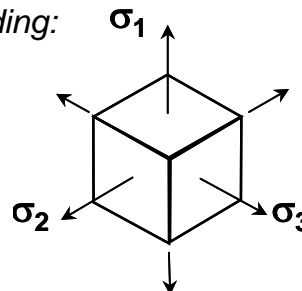
Strain is always 3D (due to Poisson's ratio).

Consider a unit cube of material (representing any volume element in a uniformly loaded body), under a general set of normal stresses ($\sigma_1, \sigma_2, \sigma_3$).

Axes:

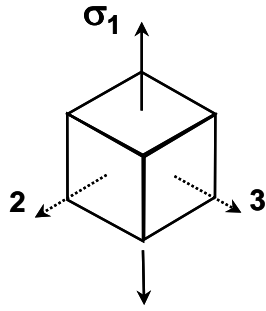


Loading:



To find resulting strains: apply each stress in turn, and use *superposition*.

e.g. load in 1-direction only:



Strains due to stress σ_1 : $\varepsilon_1 =$

$\varepsilon_2 =$

$\varepsilon_3 =$

Repeat for each stress in turn, and sum the strains:

$$\varepsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2 - \nu \sigma_3)$$

$$\varepsilon_2 = \frac{1}{E} (-\nu \sigma_1 + \sigma_2 - \nu \sigma_3)$$

$$\varepsilon_3 = \frac{1}{E} (-\nu \sigma_1 - \nu \sigma_2 + \sigma_3)$$

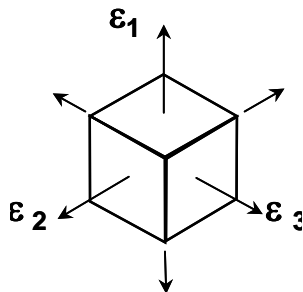
General form of Hooke's Law in 3D.

Dilatation

When materials strain elastically, their volume changes.

Volumetric strain is called the *dilatation*, $\Delta =$

Consider the unit cube again, for a general strain state ($\varepsilon_1, \varepsilon_2, \varepsilon_3$):



Initial volume: $V_0 = 1$

Final cube dimensions: $(1+\varepsilon_1, 1+\varepsilon_2, 1+\varepsilon_3)$

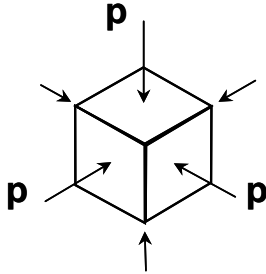
Hence final volume:

Hence for small strains ($\varepsilon \ll 1$), dilatation is:

Bulk Modulus

A state of *hydrostatic stress* is when all three normal stresses are equal, e.g. under uniform external pressure p : $\sigma_1 = \sigma_2 = \sigma_3 = -p$

This loading occurs in ceramic manufacturing, when powders are compacted under large pressures (“hot isostatic pressing, or HIPing”).



From Hooke's law in 3D (noting the symmetry):

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \frac{1}{E} (-p + \nu p + \nu p) = -\frac{p}{E}(1 - 2\nu)$$

Hence the dilatation is:

Define the *bulk modulus*, $K =$
(units: GPa)

Comparison with Young's modulus, E :

crystalline materials (metals, ceramics), $\nu \approx 0.3$:

rubber, $\nu \approx 0.5$:

2.2.3 Shear Stress and Strain

Shear Stress

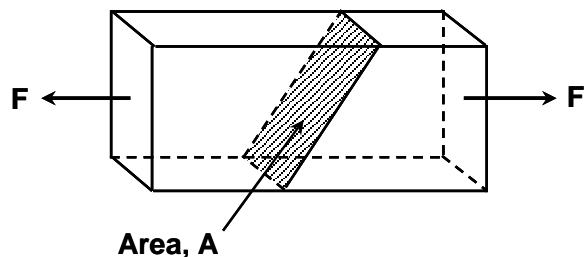
Normal stress (as above):

force per unit area carried *perpendicular* to a plane within the material.

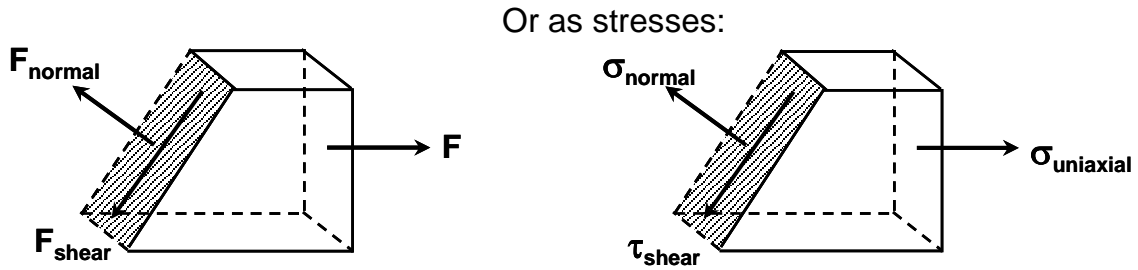
Shear stress:

force per unit area carried *parallel* to a plane within the material.

For uniaxial tension, consider an arbitrary plane which is inclined to the specimen axis:



For equilibrium, the shaded surface must carry components of force both normal and parallel to the surface:



Notes:

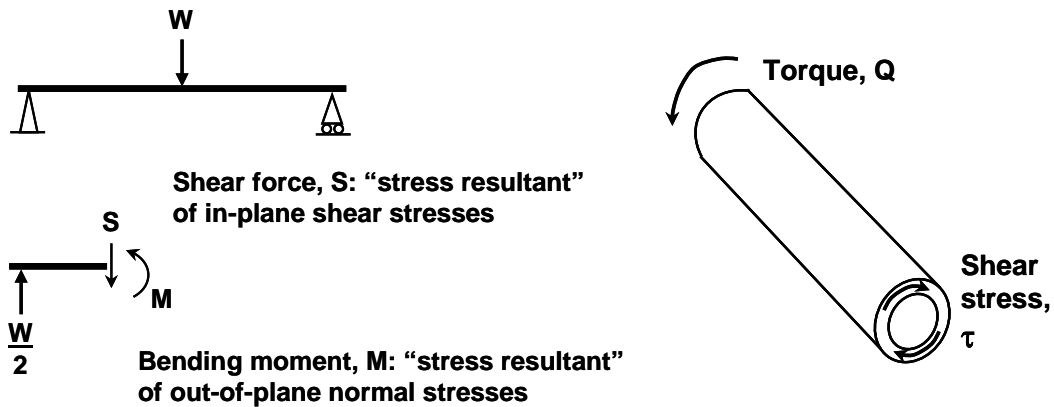
- usual symbol for *shear stress*: τ (Greek "tau");
- forces may be related by resolving, but this is NOT true for the stresses (as stresses on the inclined plane act on a different area to the axial stress);
- there is a simple trigonometric relationship between the stresses (allowing for the effect of the angle of the plane on both force components and plane area – further analysis in IB Structures).

Key points:

- the description of a stress state depends on the orientation of the surfaces “cut” through the solid;
- shear stresses arise in almost every loading situation, but may be off-axis from the obvious orientations chosen.

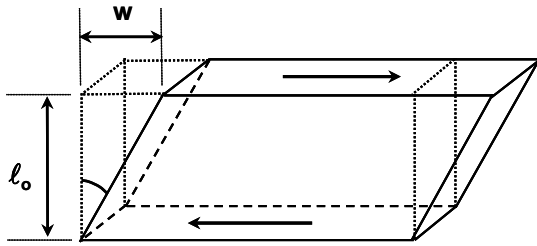
Examples of shear forces and stresses in structural components:

- arches, beams in bending, shafts in torsion



Shear strain

Shear stresses distort the *shape* of a volume element, rather than changing its axial dimensions.

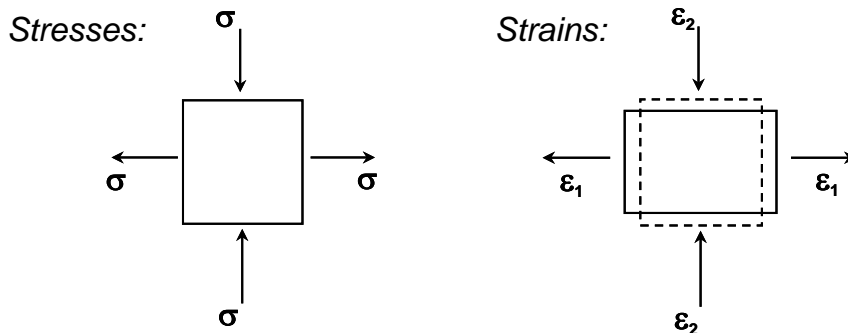


Shear strain:

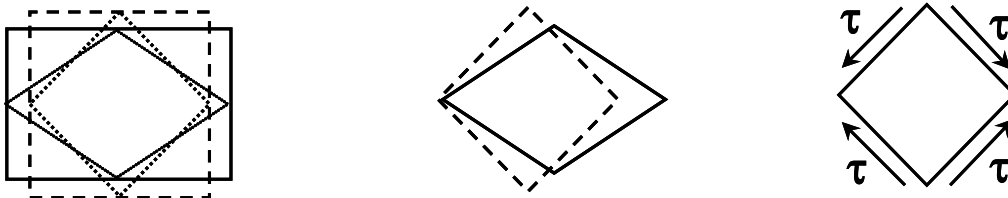
(i.e. for small strains, $\tan \gamma = \gamma$)

As for stress, the description of the strain state depends on the axes chosen.

Consider the 2D element loaded as shown: the strains will be purely normal.



Imagine the lines joining the mid-points of the original square, before and after the deformation:



Simple analysis relates:

- the pure shear stress τ to the biaxial tension/compression stress σ
- the shear strain γ to the normal strains, ϵ_1 and ϵ_2

Shear Modulus

In some loading situations, the stiffness in shear is relevant (e.g. twisting of a shaft loaded in torsion). The *shear modulus G* is defined as:

Using the normal–shear relationships between stress and strain (for pure shear) and Hooke’s Law, the *shear modulus G* and *Young’s modulus E* are related by:

Typical value for crystalline materials ($\nu \approx 0.33$):

2.2.4 Summary: Relationships between elastic constants

(i) Four elastic constants have been defined: *E*, ν , *K* and *G*.

The relationships between them are:

$$K = \frac{E}{3(1 - 2\nu)} \quad \text{and} \quad G = \frac{E}{2(1 + \nu)}$$

(ii) 3D elastic analysis assumes that the material is *isotropic* (i.e. same properties in all directions):

- OK for crystalline materials and some polymers;
- not for aligned materials: drawn polymers, wood or fibre composites.

(iii) For isotropic materials there are only *two independent elastic constants* – given any two, the others can be found.

2.3 Analysis of Stress and Strain

Analysis of stress and strain is fundamental to Structures, Materials and Mechanical Design.

The Part I Structures courses cover many in depth:

- truss deflection and failure, pressure vessels, beam stresses and deflections, torsion of shafts

These solutions are used in Materials for *material selection* problems.

Two additional situations are addressed here:

- *constrained deformation*
- *thermal stress and strain*.

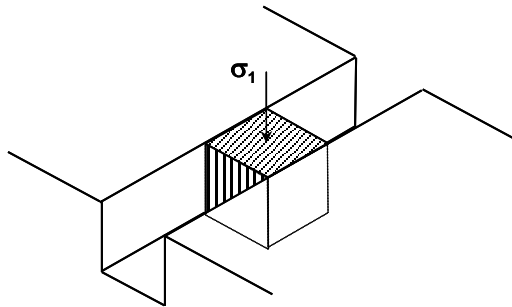
2.3.1 Constrained Deformation

Under uniaxial tension (or compression) elastic materials strain laterally (Poisson's ratio).

What if the material is constrained: i.e. lateral strain is prevented?

Example: constrained compression of a cube

Consider a cube of material fitted into a square-section slot in a rigid plate, and loaded with a compressive stress σ_1 .



2-direction: parallel to the slot
3-direction: across the slot

Strain in the 3-direction is zero:

The material tries to expand in the 3-direction – a *compressive stress* σ_3 is induced to prevent this. From Hooke's Law in 3D:

Hence:

Strain in the 1-direction (due to both stresses) is given by:

Hence the "effective modulus", $\frac{\sigma_1}{\epsilon_1} =$

Key points:

- constraint induces a *transverse stress*: nominally uniaxial loads generate biaxial (or triaxial) stresses in the material;
- the apparent material stiffness is increased, by a factor $1/(1 - \nu^2)$.

How much is this factor?

- crystalline materials ($\nu \approx 0.33$):
- rubber ($\nu \approx 0.5$):

The effect is even more marked in rubber if there is constraint in *both* transverse directions (since rubber is *incompressible*).

e.g. *shoe soles*:

- thin layers of rubber loaded in compression
- constrained (by friction) from expanding sideways
- solid rubber soles: feel very stiff (unexpected when rubber is chosen for its low Young's modulus, to provide cushioning!)

How are shoe soles designed to give a springy response (Ex. Paper 2)?

2.3.2 Thermal Stress and Strain

All materials expand as temperature rises. To a good approximation, the strain caused by a temperature change, ΔT , is proportional to ΔT .

Strain/unit temperature change = *thermal expansion coefficient*, α (units: K^{-1}).

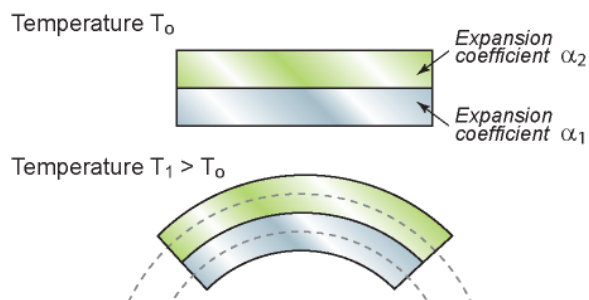
Typical values:

	Material	$\alpha \times 10^{-6} (K^{-1})$
Ceramic	Alumina	9.0
Metals	Steels	12
	Aluminium Alloy	22
	"Invar" (Ni-Fe alloy)	1.0
Polymer	Nylon	145

Thermal expansion is important in design when:

- *constrained expansion or contraction* (inducing *thermal stresses*);
- *temperature gradients* exist across a product, again inducing thermal stresses (e.g. heat treatment of metals);
- *dissimilar materials* are joined and then subjected to temperature change, giving *differential thermal expansion/contraction*, and induced stresses.

The last of these is exploited directly in the *bimetallic strip*:



Example 1: railway track

Estimate the stress induced in a continuous steel railway track subjected to a 40°C temperature rise above an initial (stress-free) temperature.

Thermal strain $\epsilon_{\text{thermal}} =$

Total strain = 0, hence: $\epsilon_{\text{elastic}} = -\epsilon_{\text{thermal}}$

Induced elastic stress:

Values for steel: $\alpha = 12 \times 10^{-6} \text{ K}^{-1}$, $E = 210 \text{ GPa}$

For $\Delta T = 40 \text{ K}$:

In compression this stress may cause *buckling*.
Hence must leave expansion gaps, or install under tension.

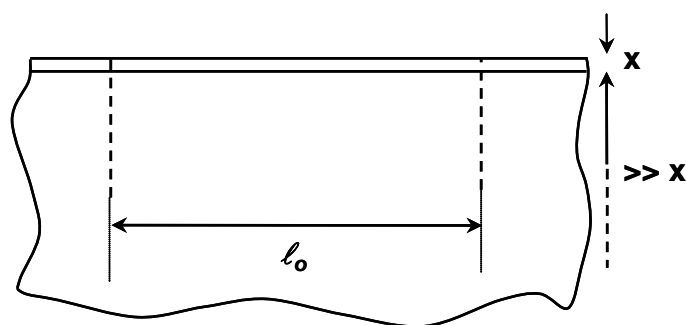
Adelaide, January 2009



Example 2: constrained surface layers

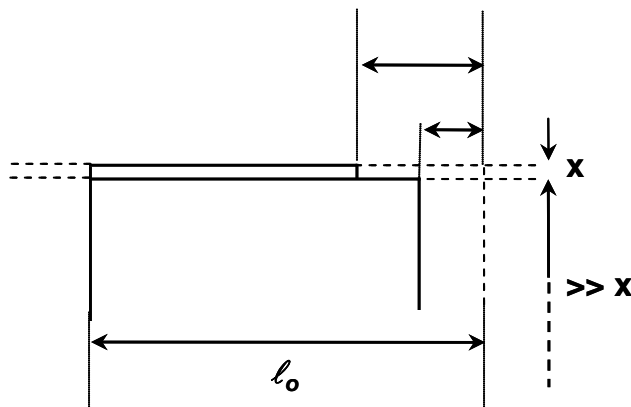
Many coating technologies (for *wear or corrosion resistance*) deposit a thin surface layer of a different material on a component at high temperature.

Consider a thin film on a component of thickness \gg film thickness, x , and examine a length l_0 (at high temperature) away from the edges of the specimen:



On cooling, *differential contraction* of substrate and surface layer can induce *thermal stresses*.

First imagine the layer is *not* attached to the substrate, and cool component by a temperature drop ΔT :



For $\alpha_1 > \alpha_2$, surface layer is restrained from contracting by the substrate and goes into *tension*, balanced by *compression* in the substrate.

As the surface layer is thin compared to the substrate:

- compressive stress negligible (large area to provide balancing force);
- final length = that of substrate =

Superpose tensile stress in surface layer, to increase its length from its contracted length, to match that of the substrate:

- change in length:
- strain in surface layer:

Since $(\alpha_1 \Delta T) \ll 1$, strain =

Notes:

- if $\alpha_1 < \alpha_2$, analysis still valid, but surface layer goes into *compression*.
- surface stress is *biaxial* (and equal in all directions): use 3D Hooke's Law to find biaxial stress in surface layer that produces this strain (Ex. Paper 2).
- key result is:

2.4 Measurement of Young's Modulus

Wide range of methods for measuring Young's modulus:

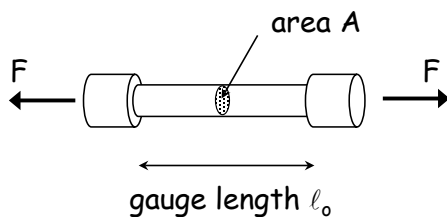
- tensile testing (Experiment 4)
- bending stiffness of a beam
- natural frequency of vibration (beams, plates – IB Experiment M1)
- speed of sound in the material, $\sqrt{E/\rho}$

(Notes below on practicalities, accuracy etc.)

Tensile Testing:

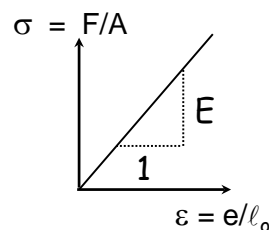
Uniaxial tension applied to sample of standard, uniform cross-section:

e.g. Hounsfield tensometer (Materials Lab, Experiment 4)



Measure extension of gauge length, from machine extension, or directly from the sample using a *strain gauge*.

Convert load vs. extension graph to stress vs. strain: **gradient E**.

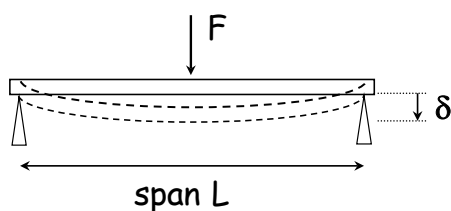


Accuracy:

- elastic extensions small ($< 0.1\%$), difficult to measure precisely
- measurement from machine must allow for flexure of machine
- strain gauge much higher precision

Bending Stiffness of a Beam:

Beam of uniform cross-section loaded in 3-point bending:



Stiffness (F/δ) related to E via beam theory (IA Structures).

$$\text{From Structures Databook: } \delta = \frac{F L^3}{48 E I}$$

EI is the **flexural rigidity**; I is the *second moment of area*, and captures the effect of cross-section size and shape:

$$\text{e.g. for a square section of side-length } D: I = D^4 / 12$$

$$\text{Hence for 3-point bend of square section beam: } E = \frac{(F/\delta)L^3}{4D^4}$$

Accuracy:

- bending gives much more deflection for given load than tension
- E sensitive to L and D : requires accurate measurement of dimensions

Natural frequency of vibration (beams and plates)

Natural frequency of vibration in bending depends on *flexural rigidity, EI*

Beam supported at nodal points and set vibrating:

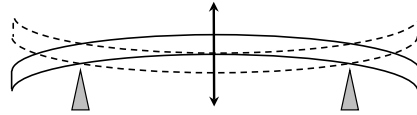


Plate supported on soft mounts over a loudspeaker emitting note of adjustable frequency.

“Chladni patterns” – nodal lines (revealed using tea leaves)

(Note: this method is also a good way to measure *Poisson’s ratio*)



For video demo, see: http://www-materials.eng.cam.ac.uk/HRSMaterials/chladni_demo.html

Accuracy:

- measuring frequency more accurate than deflection
- calculated E still sensitive to beam/plate dimensions (through I and L)

Speed of Sound in the material:

The speed of sound v_t in a solid depends on stiffness of inter-atomic bonds.

Young’s modulus E also depends on bond stiffness (see later notes).

Hence measure E by measuring v_t :

- strike a bar of material on one end
- time the longitudinal wave reflected from far end of the bar

Dimensional analysis: $E = v_t^2 \rho$, where ρ is the density

i.e. speed of sound $v_t = \sqrt{E/\rho}$

e.g. for steel: $E = 210$ GPa, $\rho = 7800$ kg/m³

$$v_t = (E/\rho)^{1/2} = (210 \times 10^9 / 7800)^{1/2} = 5188 \text{ m/s}$$

Accuracy:

- for 1m long bar, time between pulses $\approx 1/5000 = 0.2$ ms
- accuracy depends on precise time measurement, which is relatively easy with piezoelectric transducers

2.5 Data for Young's Modulus – Material Property Charts

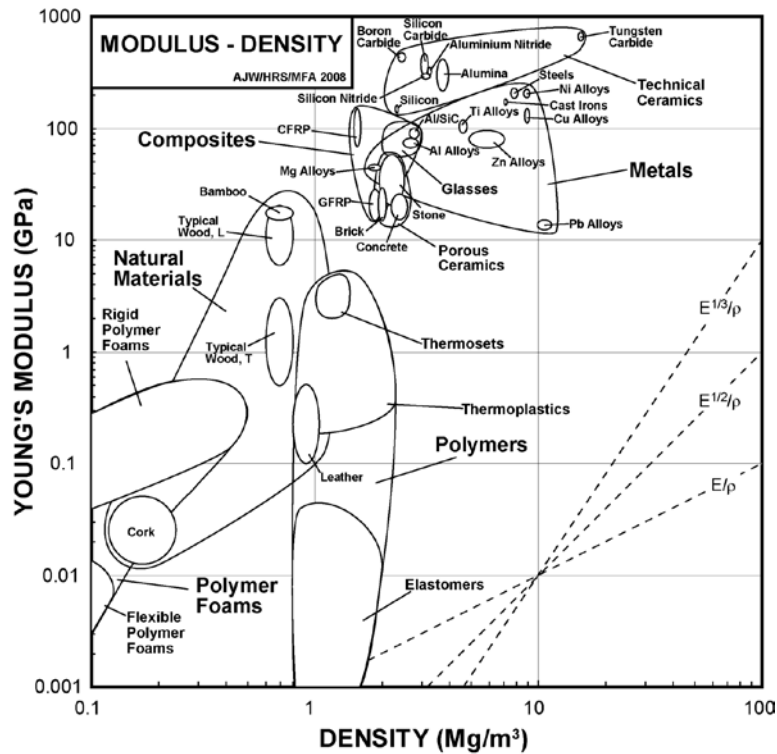
Data sources for Young's Modulus (and other properties):

- *Materials Databook*
- *Cambridge Engineering Selector* (CES) databases

Material Property Charts

- graphs with two properties as axes
- facilitates material comparison and making trade-offs in design

Material Property Chart: Young's Modulus vs. Density (Materials Databook)



Notes on Property Charts:

- log scales: properties span many orders of magnitude
- ranges of the two properties define a “bubble” on the chart (usually drawn as an ellipse)
- materials cluster by class, enclosed by an envelope

Notes on Young's modulus:

- Young's modulus for solid materials spans over 6 orders of magnitude
- metals, ceramics and composites are stiff materials
- polymers, natural materials and foams are generally much less stiff, and cover a very wide range of stiffness

Notes on Density:

- density for solid materials spans a factor of 20
- polymer foams & porous natural materials (e.g. wood) extend the range to much lower values
- most ceramics, and composites, have similar densities to the “light metals” (Mg, Al, Ti); steels and other metals have high densities
- only porous materials and the lightest polymers float in water ($\rho = 1 \text{ Mg/m}^3$)

Physical observations:

- materials in a given class (metals, ceramics etc) cluster together (indicating similarity in underlying physical basis of the properties)
- general trend for ceramics and metals: Young's modulus increases with density (though with exceptions, such as Pb alloys)
- individual classes of metals (e.g. steels, Ti alloys etc) have well-defined values of Young' modulus and density (small bubbles)
- polymers cover a wide range of Young's modulus, but their densities are all around 1-2 Mg/m³; individual polymers are omitted for clarity

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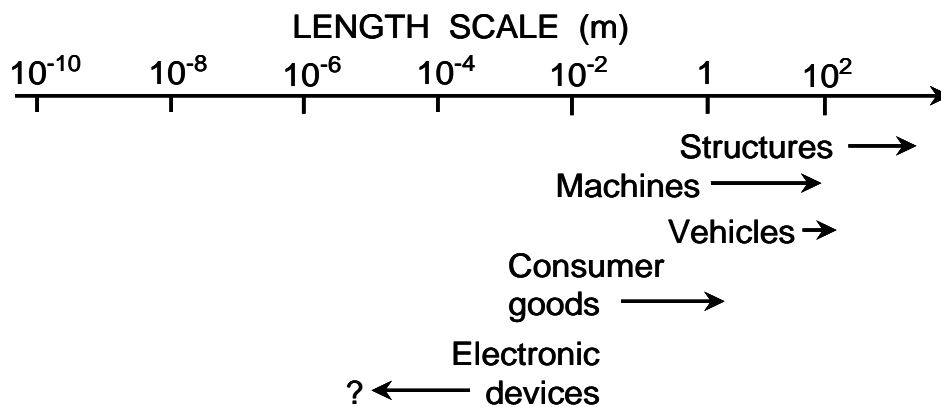
Section 3 covers Examples Paper 2, Q.7-10.

3. Microstructure of Engineering Materials I

3.1 Introduction: Length-scales in engineering materials

Microstructure for Engineers

Engineering spans length scales from kilometre-scale structures to electronic devices and micro-electro-mechanical-systems (MEMS) at fractions of 1 mm.



Atoms: typical radius = 0.1 – 0.2 nm

Microstructure: from atom-scale defects, compounds of atoms, “grains” (crystal regions), to “engineering” defects such as cracks, porosity and surface roughness (at μm – mm scale).

“Teach Yourself Microstructure”

Topics covered (and assumed in lectures):

- primary and secondary bonding
- crystal packing in metals and ceramics, and atomic basis of density
- polymer microstructure

Why do engineers need to know about microstructure?

- macroscopic engineering-scale **properties** (mechanical, thermal, electrical...) are governed directly by the **internal microstructure**;
- different features at different **length-scales** determine each property.

Understanding microstructure enables materials engineers and scientists to:

- recognise the physical origin and inherent limits on properties, i.e. to target best openings for developing new or improved materials;
- know what *can* be manipulated (e.g. by changes in **composition**, or in **processing**) and what *cannot*;
- avoid key **failure mechanisms** in design: yield, fracture, corrosion.

3.2 Crystalline Materials: Metals, Ceramics

In *crystalline materials*, atoms pack in a regular repeating lattice structure.

Metals are sometimes used in a **pure form** (e.g. Al foil, Cu conductors) but most commonly as **alloys** (mixed with other metal and non-metal elements):

- **Steels and cast irons** (Fe-C, plus Mn, Ni, Cr....)
- **Al alloys** (Al+Mg, plus Cu, Zn, Si....)
- **Cu alloys** (Cu-Sn: bronze; Cu-Zn: brass)

Ceramics are **compounds** of metals or silicon with non-metals (O, C or N):

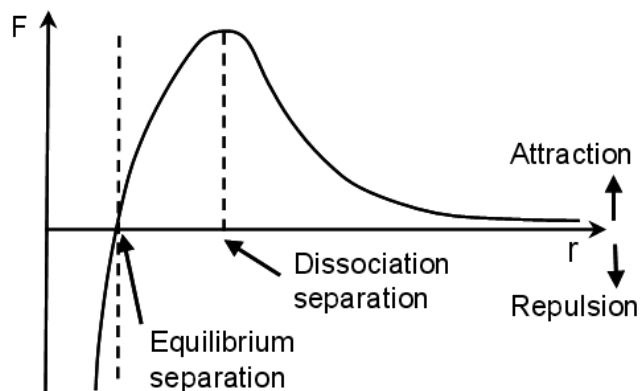
- **Technical ceramics**: alumina, silicon carbide, silicon nitride
- **Glasses** (based on silica)
- **Porous ceramics**: brick, concrete, pottery

Properties and applications of common metals and alloys, and ceramics, are tabulated in the *Materials Databook* (and in the *CES software*).

3.3 Atomic Basis of Elasticity of Metals and Ceramics

Elastic Response of Metals and Ceramics

Primary bonds behave as stiff elastic springs, with force–displacement response of the form:



Gradient of the F– r response (at the equilibrium spacing r_o) is the **bond stiffness** S_o :

$$S_o = \left(\frac{dF}{dr} \right)_{r=r_o}$$

For a *small* displacement from equilibrium, u :



Restoring *force per atom* =
and the *strain* =

Number of bonds/unit area is fixed (by the lattice geometry)

Area per bond is of order r_0^2

Hence the *stress* \approx

i.e. Young's modulus, $E = \text{stress/strain} \approx$

Key points:

- Young's modulus of a component directly reflects the bond stiffness, on a length scale 10^{10} times smaller!
- atomic $F-u$ response is linear, giving linear elasticity at the macro scale;
- primary bond stiffness and atomic packing are physically prescribed;
- *little scope for manipulating Young's modulus* via the crystal structure;
- recall that the same is true for *density* (*Teach yourself microstructure*).

Properties of Alloys

Alloys are mixtures of elements, forming *solid solutions* and *compounds*.

Density

From the hard sphere model, solid solutions and compounds will form densities between those of the pure elements.

Example densities (Mg/m^3):

Young's Modulus

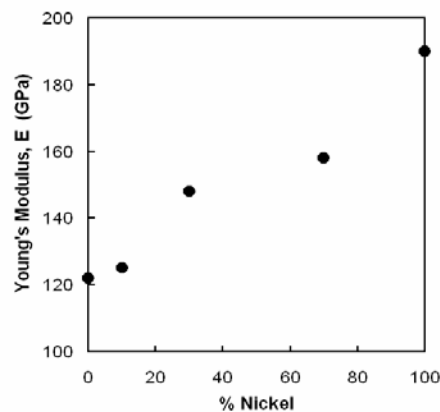
Solid solutions contain a mixture of different *bond stiffnesses* (A-A, A-B, B-B):
 \Rightarrow Young's modulus of A-B solutions somewhere between pure A and pure B.

Compounds have *stiffer* bonds, and higher modulus – the stronger chemical bond is a major reason why the compound forms.

Example: **Copper-nickel alloys** form solid solutions of Cu and Ni across *all* compositions (used for conductors, electroplating, coinage).

Data for E across the composition range (from CES):

Alloy	E (GPa)
Pure Cu	122
Cu – 10% Ni	125
Cu – 30% Ni	148
Cu – 70% Ni	158
Pure Ni	190



Summary: key points

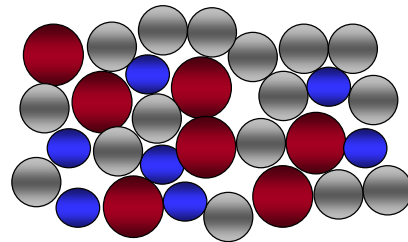
- Young's modulus and density of crystalline materials are governed directly by atomic packing and atomic bonding;
- For alloys, both properties are well-defined by the composition, with no scope for change by processing.

A short aside: amorphous metals

Metals crystallise easily – some can be forced to retain an **amorphous structure** if cooled very fast.

Recently, **bulk amorphous metals** have been produced which solidify at conventional rates, using very unusual compositions:

e.g. Be–Zr–Ti–Cu–Ni–Y alloy!



These alloys contain atoms of *widely different sizes*, making regular crystal packing difficult.

Amorphous metals have unusual properties:

- mechanically and magnetically hard
- very low damping (little energy lost in elastic collisions).

3.4 Polymer Elasticity

Classes of polymers

Thermoplastics: polyethylene (low & high density: LDPE, HDPE), polyvinylchloride (PVC), polypropylene (PP), polystyrene (PS)
Trade-names commonly used: nylon, acrylic, teflon, perspex...

Elastomers: rubbers, neoprene

Thermosets: epoxies, phenolics, polyurethane

Polymer nomenclature, properties and typical applications are summarised in the Materials Databook and the CES software.

Polymer “Alloys”

Polymer molecular chemistries are well-defined, and different polymers don't mix as freely as metallic atoms.

There are two mechanisms for “alloying” polymers:

Copolymers: more than one monomer polymerised together – only a few combinations will do this, e.g. acrylonitrile, butadiene, styrene (ABS).

Polymer blends: molecular-scale mixtures of two polymer chains, without cross-linking.

Glass transition, melting and decomposition temperatures

In **crystalline materials and glasses**, the breaking of *primary* bonds by thermal energy gives a well-defined **melting point**, T_m .

In **polymers**, the weaker *secondary* bonds are overcome by thermal energy at a lower temperature: the **glass transition temperature**, T_g .

Typical values for T_g
(see Materials Databook):

Polymer	T_g (°C)
Acrylic (PMMA)	85 - 165
Polystyrene (PS)	74 - 110
Polyethylene (PE)	- 25 - - 15
Natural Rubber	- 78 - - 63

Compare with melting points T_m of primary bonded crystalline materials:

- metallic bond:
- ionic bond:
- covalent bond:

Above T_g , the behaviour differs between the polymer classes:

- **Amorphous thermoplastics**: melt to a viscous liquid (entangled molecules slide over one another).
- **Semi-crystalline thermoplastics**: amorphous regions melt, crystalline regions survive to a higher *melting point*, T_m (typically $T_m \approx 1.5 T_g$, in K), above which a viscous liquid forms.
- **Elastomers** and **thermosets**: secondary bonds melt at T_g but cross-links do not – on heating the polymer does not melt, but decomposes or burns.

Consequences for **processing** and **environmental impact**:

Thermoplastics:

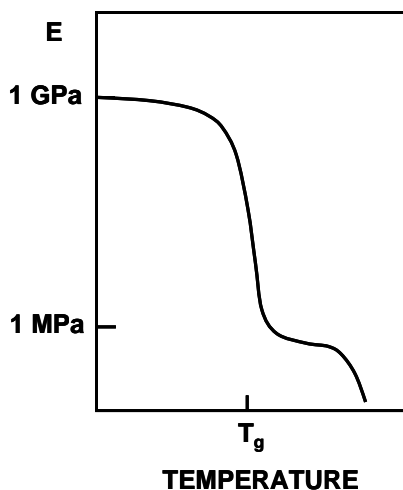
Elastomers/Thermosets:

Elastic Response of Polymers

Elasticity of polymers:

- is sensitive to **temperature** (relative to T_g) and **rate of loading**
- differs between polymer classes, due to **crystallinity** and **cross-linking**.

(1a) Amorphous thermoplastics



Glassy region ($T < T_g$):

- loading stretches low stiffness secondary bonds
- E of order 1–3 GPa
(cf: metals, 40–200 GPa;
ceramics, 100–1000 GPa)

Glass transition ($T \approx T_g$):

- segments of chain melt, and modulus falls steeply
- *reptation* takes place – molecules slide past one another
- transition not abrupt (range of bond lengths in amorphous microstructure)

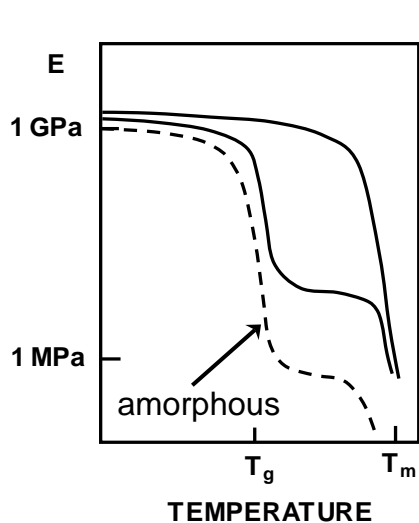
Rubbery region/viscous flow ($T > T_g$):

- very low stiffness above T_g (roughly 1000 times lower: a few MPa)
- rubbery elasticity is due to entanglement points
- above $1.4 T_g$, chains all slip: viscous flow

Footnote – effect of loading rate:

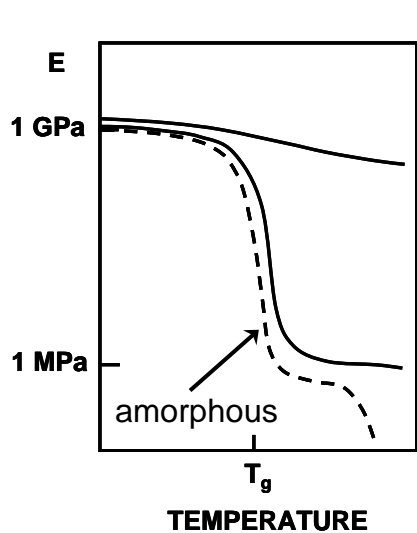
- deformation around and above T_g relies on molecular sliding, which is sensitive to the rate of deformation
- hence E depends on the loading rate
- T_g must be defined at a reference loading rate

(1b) Semi-crystalline thermoplastics: effect of crystallinity



- Crystalline regions stiffer than amorphous: higher E below T_g .
- Glass transition: no effect on crystalline regions.
- Rubbery region: E increases with degree of crystallinity
- At high crystalline fractions, glass transition has no effect on E .
- At T_m , crystalline regions melt: viscous flow.

(2,3) Elastomers, thermosets: effect of cross-linking



degree of cross-linking:

Elastomers:

- Below T_g (well below room temperature): glassy
- Above T_g : pronounced rubbery region.

In rubbery region:

- large recoverable strains at low stresses
- chains unravel extensively, pulled back by cross-links when the load is released.

Thermosets:

- Highly cross-linked, no effect of glass transition.
- Stiffer than thermoplastics in the glassy region; E falls slowly on heating.

Summary – key points:

- Young's modulus and density of polymers are governed directly by molecular packing and bonding;
- elastic response is sensitive to detail in molecular structure, in particular the degree of crystallinity or cross-linking (defining the polymer classes);
- significant drop in modulus in thermoplastics and elastomers at the glass transition (which itself depends on the rate of loading);
- both E and ρ are much lower than for metals and ceramics;
- Young's modulus of polymers can be manipulated more than in metals (by changing molecular weight, polymer chemistry, and degree of crystallinity or cross-linking – via processing).

3.5 Manipulating Properties I

3.5.1 Foams

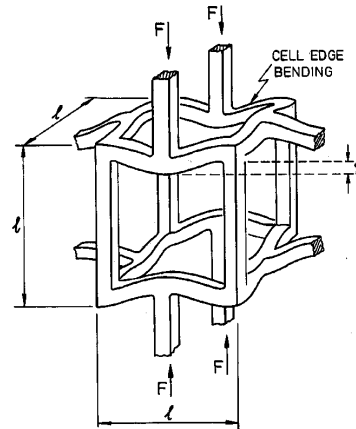
Foams are *porous solids*. Porous, cellular solids are found extensively in nature – wood is a highly efficient natural engineering material.

Commercial foams use polymers of various stiffnesses, with open or closed cells. Recently, ceramic and metallic foams have been developed.

Foams are described by their *relative density* = $\frac{\text{density of foam}}{\text{density of solid used}}$

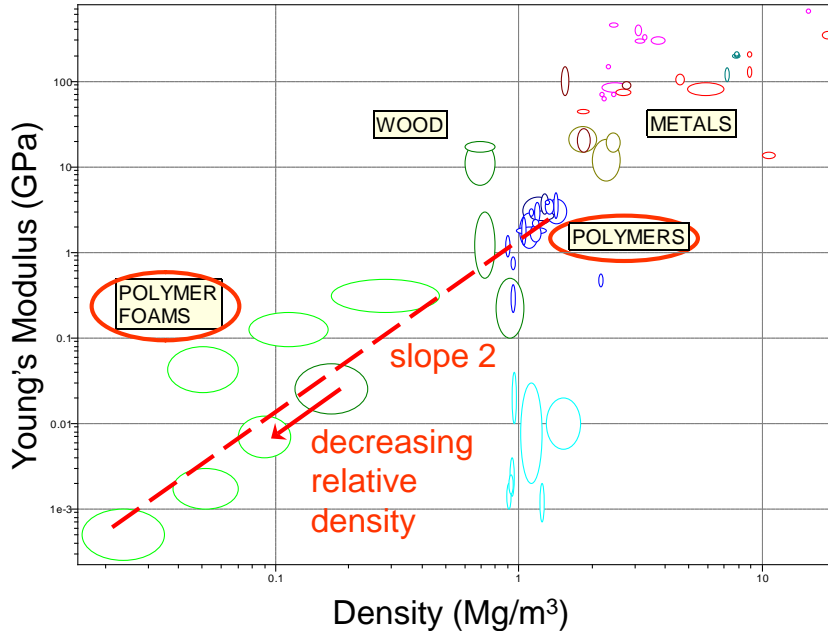
Simple model for *Young's modulus of a foam*:

- idealised unit of material as shown
- elastic response dominated by *bending* of the solid ligaments



Key result is:
$$\frac{E_{foam}}{E_{solid}} = \left(\frac{\rho_{foam}}{\rho_{solid}} \right)^2$$

On Young's modulus–density chart: lines of slope 2 lead from solid to foam.



The theory works well, enabling prediction of E, ρ of “new” materials, e.g. *aluminium foams*

3.5.2 Composite materials

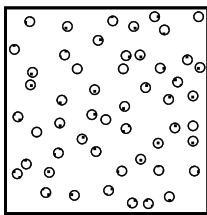
Composite materials combine two materials to produce *new property profiles*, exploiting separate qualities of the individual components.

Example: **CFRP** (carbon fibre reinforced polymer)

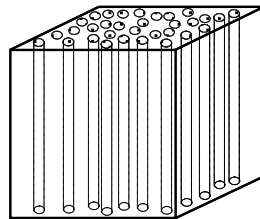
Material	E (GPa)	ρ (Mg/m ³)	E/ ρ	Comment
Epoxy	2.7	1.25	2.2	Low specific stiffness
C fibres	400	1.85	216	Very high specific stiffness, but not in a usable structural form
CFRP	69-150	1.55	45-100	Fibres + epoxy provides usable form, and protects fibres from damage
Structural alloys	42-210	1.8-9.0	≈ 23	CFRP superior in specific stiffness to all alloys

Three main *composite geometries*, dependent on component architecture:

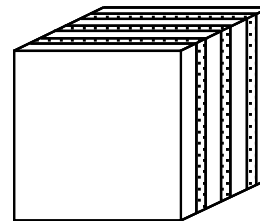
- **Particulate:** metal – ceramic: Al-SiC, WC-Co;
polymer – ceramic: “filled polymers”, e.g. glass-polyester
- **Fibres:** carbon/glass/Kevlar fibre – polymer
- **Laminates:** plywood; “GLARE”, “ARALL” (Al – GFRP laminates)



Particulate



Fibre



Laminate

Composite processing:

Particulate composites:

- add micron-scale particles to melt before casting or moulding
(Note that many polymers are powder-filled – to provide stiffening, colour, UV resistance etc.)

Fibre composites:

- short chopped fibres: mix with resin, shape in a mould, e.g. canoes
- long continuous fibres:
 - lay out fibre mats in a mould, and infiltrate with resin
 - combine fibre+resin in “*prepreg*” plies, then laminate

Laminates:

- wood etc: stack and glue thin layers
- long-fibre composites: stack multiple layers of prepreg with different fibre orientations, hot form to shape, curing the resin

Estimates for Composite Properties

Composites are defined by the **volume fraction** V_f of one of the components, e.g. the volume fraction of the particle or fibre reinforcement added to a matrix material.

Density

For all composites, density follows a simple **rule of mixtures** between the densities of the two components (i.e. mass conservation):

$$\rho_c = V_f \rho_f + (1 - V_f) \rho_m$$

Note subscript notation:

c = composite, m = matrix, f = reinforcement (as often *fibres*)

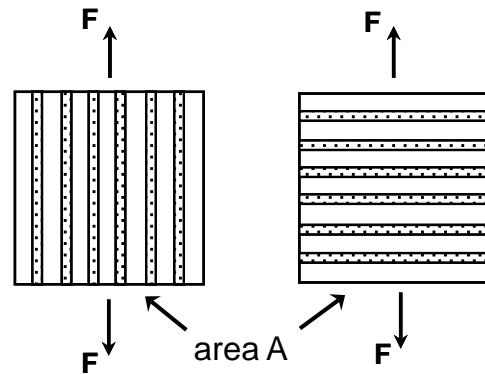
Young's Modulus

Laminated composite, in edge view:

Layers have modulus E_f and E_m
($E_f > E_m$)

V_f = volume fraction of stiffer (E_f)
material.

Stiffness differs parallel and perpendicular to the layers – the material is **anisotropic**.



Loading **parallel** to the layers:

Loading **perpendicular** to the layers:

Young's Modulus **parallel** to layers (for derivations – see Appendix)

Longitudinal modulus:
$$E_c = \frac{\sigma_c}{\epsilon_c} = V_f E_f + (1 - V_f) E_m$$

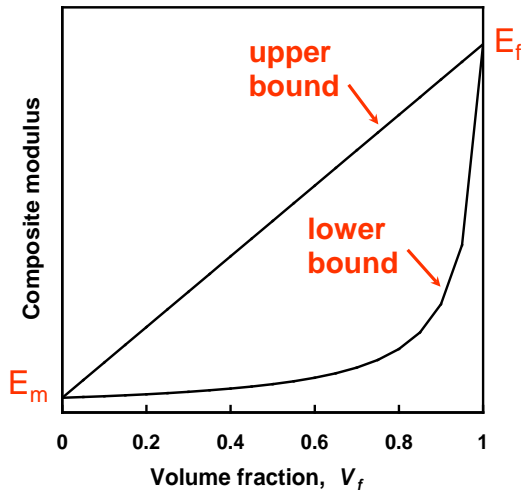
(denoted $E_{||}$ in Databook)

Young's Modulus **perpendicular** to layers

Transverse modulus:
$$E_c = \left(\frac{V_f}{E_f} + \frac{(1 - V_f)}{E_m} \right)^{-1}$$

(denoted E_{\perp} in Databook)

These “Voigt-Reuss equations” are **upper and lower bounds** for E : composite moduli must lie between (or on) these limits.



Particulate composites ?

- isotropic (same in all directions)
- close to lower bound (equal stress)

Fibre composites ?

- anisotropic (stiffer parallel to fibres)
- parallel to fibres: upper bound exact (equal strain)
- perpendicular to fibres: close to lower bound (equal stress)

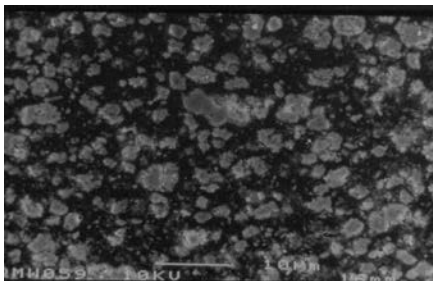
Note: practical upper limit on V_f in fibre and particulate composites, $\approx 50\%$.

Case study: artificial bone for prosthetic implants

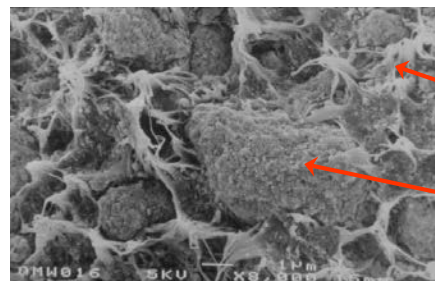


Problem: Prosthetic hip implants – Ti alloy or stainless steel (15-30 x stiffer than bone). Load transfer can be poor – the bone *resorbs* under too much stress.

Possible solution: Particulate composite – **HDPE + hydroxyapatite**, HA (the mineral found in bone, but manufactured artificially).



Prototype HDPE-HA composite



Fracture surface

Images courtesy:
E. Tanner, QMC London

HDPE

HA

- good *biocompatibility*
- potentially close *match with E for bone* (see Examples Paper 2)

Appendix: Derivation of composite Young's modulus *parallel* to layers

Total load F = load in f layers + load in m layers = $A_f \sigma_f + A_m \sigma_m$

where A_f, A_m = cross-sectional area of f and m layers, respectively

Average stress in composite, $\sigma_c = F/A = \frac{A_f}{A} \sigma_f + \frac{A_m}{A} \sigma_m$

But area fractions of f and m layers: $\frac{A_f}{A} = V_f$ and $\frac{A_m}{A} = (1 - V_f)$

Hence stresses follow a rule of mixtures: $\sigma_c = V_f \sigma_f + (1 - V_f) \sigma_m$

Substitute for stresses using Hooke's Law: $\sigma_f = E_f \varepsilon_f, \sigma_m = E_m \varepsilon_m$

Hence: $\sigma_c = V_f E_f \varepsilon_f + (1 - V_f) E_m \varepsilon_m$

But strains same in composite and each material: $\varepsilon_c = \varepsilon_f = \varepsilon_m$

Longitudinal modulus: $E_c = \frac{\sigma_c}{\varepsilon_c} = V_f E_f + (1 - V_f) E_m$

(denoted E_{II} in Databook)

Derivation of composite Young's Modulus *perpendicular* to layers

Similar argument followed for the *transverse modulus*

- stress the same in both materials
- V_f and $(1 - V_f)$ correspond to "length fractions" in loading direction
- *total strain* found by noting that total *extension* is the sum of the extensions in the two materials (i.e. *strain* follows a rule of mixtures)

Complete this derivation as an exercise (for worked solution – see online resources).

Transverse modulus: $E_c = \left(\frac{V_f}{E_f} + \frac{(1 - V_f)}{E_m} \right)^{-1}$

(denoted E_{\perp} in Databook)