Engineering Tripos Part IA   **First Year**

Paper 2 – MATERIALS

HANDOUT 2

2. **Elastic Properties of Materials**
   2.1 Elastic Stiffness in Design
   2.2 Stress, Strain, Hooke's Law and Elastic Moduli
   2.3 Analysis of Stress and Strain
   2.4 Measurement of Young's Modulus
   2.5 Data for Young's Modulus – Material Property Charts

3. **Microstructure of Engineering Materials I**
   3.1 Introduction: Length Scales in Materials Engineering
   3.2 Crystalline Materials: Metals, Ceramics
   3.3 Atomic Basis of Elasticity of Metals and Ceramics
   3.4 Polymer Elasticity
   3.5 Manipulating Properties I: Foams, Composite Materials

Section 2 covers Ex. Paper 2, Q.1-6; Section 3: Q.7-10.

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2. **Elastic Properties of Materials**

2.1 **Elastic Stiffness in Design**

Many engineering components need to behave *elastically* under load, i.e.

- deformation fully recovered on unloading
  (no permanent deformation or fracture)

- maximum allowable deflection under load

Examples:

- "structural" applications: bridges, buildings, vehicles, bicycles, furniture, machines ....

- "functional" applications: precision instruments, hard disk drives ….

The *stiffness* of a component or structure is:

$$\text{Stiffness, } S = \frac{\text{Force}}{\text{Displacement}}$$

e.g. cantilever:

$$S = W / \delta$$

Stiffness depends on:
- material (steel, wood, CFRP…)
- cross-section size + shape (tube, I-beam …)
- mode of loading (tension, bending)
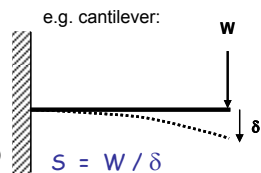
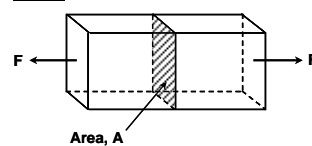Intrinsic *material* stiffness measured by *Young's modulus, E* (defined below).

Effects of size, shape, loading: use results from IA Structures.

2.2 **Stress and Strain**

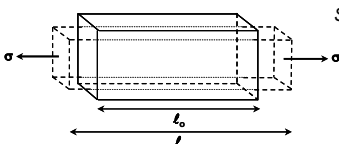2.2.1 **Uniaxial Tension and Compression**

Stress

$$\text{Stress, } \sigma = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

(tension +ve, compression -ve)

Area, A

This is a *normal* or *direct stress* (i.e. force *perpendicular* to loaded area).

Stress is measured in *Pascals* (usually MPa: 1 MPa = 10^6 N/m^2 = 1 N/mm^2).

Strain

$$\text{Strain, } \varepsilon = \frac{\text{Extension}}{\text{Original length}}$$

$$= \frac{\ell - \ell_o}{\ell_o}$$

(tension +ve, compression -ve)

This is a *normal* or *direct strain* (i.e. extension *parallel* to original length).

Strain is a ratio of lengths, so it is *dimensionless* (no units).

For most materials elastic strains are small (< 0.1%) – rubber is an exception.

Poisson's ratio

In uniaxial tension:
- the material gets longer *and* thinner
- the lateral contraction and the tensile extension are proportional
- the ratio is a material property: *Poisson's ratio*, ν (Greek "*nu*", not "v"):

$$\nu = -\frac{\text{Lateral strain}}{\text{Tensile strain}} \quad \text{(note minus sign, so } \nu \text{ is positive)}$$

Lateral strain is *not* due to volume conservation, but reflects the way atomic bonds deform under load. Typical values of Poisson's ratio, ν :

crystalline materials (metals, ceramics)	0.2 – 0.33
porous solids (foam, cork)	≈ 0
elastomeric polymers (rubber)	≈ 0.5

When does Poisson's ratio matter?

- not important in most design with *uniaxial loads*
  (e.g. members in a truss: fractional change in area)

- important when the *stress state is 2D* or *3D*
  (e.g. constrained expansion – examples later)

- important in *vibration of plates*
  (e.g. affects frequencies of vibration modes)

- important in *large strain bending,*
 giving *anticlastic curvature*:

(try bending a standard drawing rubber!)
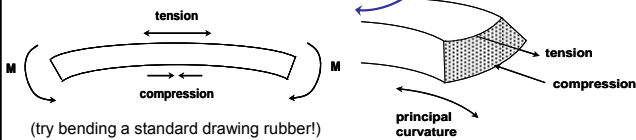
Young's Modulus

Most engineering materials are *linear elastic*: under uniaxial load, a sample of constant cross-section extends in proportion to the load (Hooke's Law).

By converting load to stress, and extension to strain:
  - the response is made independent of geometry;
  - the slope is then a material property, *Young's modulus, E*.
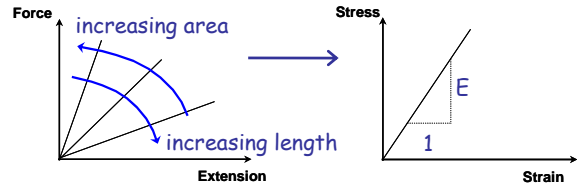
For uniaxial tension (or compression):

Young's modulus, E $= \dfrac{\text{Tensile stress}}{\text{Tensile strain}}$ (units: GPa)


2.2.2 Stress and Strain in 3D

Hooke's Law in 3D

Some engineering components are loaded uniaxially, e.g. cables, struts; but often the stresses are *multiaxial*, e.g. beams, arches, pressure vessels.
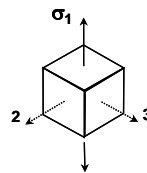
Strain is always 3D (due to Poisson's ratio).

Consider a unit cube of material (representing any volume element in a uniformly loaded body), under a general set of normal stresses (σ_1, σ_2, σ_3).

Axes: *Loading:*
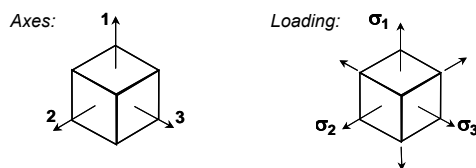
To find resulting strains: apply each stress in turn, and use *superposition*.

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e.g. load in 1-direction only:

Strains due to stress σ_1: $\varepsilon_1 = \dfrac{\sigma_1}{E}$

$\varepsilon_2 = -\dfrac{\nu\,\sigma_1}{E}$

$\varepsilon_3 = -\dfrac{\nu\,\sigma_1}{E}$

Repeat for each stress in turn, and sum the strains:

$$\varepsilon_1 = \frac{1}{E}\left(\sigma_1 - \nu\sigma_2 - \nu\sigma_3\right)$$
$$\varepsilon_2 = \frac{1}{E}\left(-\nu\sigma_1 + \sigma_2 - \nu\sigma_3\right)$$
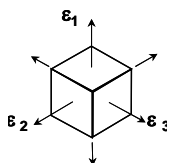$$\varepsilon_3 = \frac{1}{E}\left(-\nu\sigma_1 - \nu\sigma_2 + \sigma_3\right)$$

General form of Hooke's Law in 3D.

Dilatation

When materials strain elastically, their volume changes.

Volumetric strain is called the *dilatation, Δ* $= \dfrac{\Delta V}{V_o} = \dfrac{\text{Change in volume}}{\text{Original volume}}$

Consider the unit cube again, for a general strain state (ε_1, ε_2, ε_3):

Initial volume: $V_o = 1$

Final cube dimensions: $(1+\varepsilon_1, 1+\varepsilon_2, 1+\varepsilon_3)$

Hence final volume:

$V = (1 + \varepsilon_1) \times (1 + \varepsilon_2) \times (1 + \varepsilon_3)$

$= 1 + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

$+ \text{(higher order terms in } \varepsilon)$

Hence for small strains ($\varepsilon \ll 1$), dilatation is: $\Delta = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

Bulk Modulus

A state of *hydrostatic stress* is when all three normal stresses are equal, e.g. under uniform external pressure p: $\sigma_1 = \sigma_2 = \sigma_3 = -p$

This loading occurs in ceramic manufacturing, when powders are compacted under large pressures ("hot isostatic pressing, or HIPing").
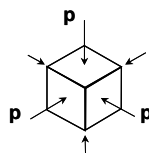
From Hooke's law in 3D (noting the symmetry):

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \frac{1}{E}\left(-p + \nu p + \nu p\right) = -\frac{p}{E}(1 - 2\nu)$$

Hence the dilatation is: $\Delta = \dfrac{-3p\,(1 - 2\nu)}{E}$

Define the *bulk modulus, K* $= \dfrac{\text{Hydrostatic stress}}{\text{Volumetric strain}} = \dfrac{E}{3(1 - 2\nu)}$
(units: GPa)

Comparison with Young's modulus, E:
  crystalline materials (metals, ceramics), $\nu \approx 0.3$: $K \approx E$

  rubber, $\nu \approx 0.5$: $K \to \infty$!! (i.e. rubber is incompressible)

2.2.3 Shear Stress and Strain
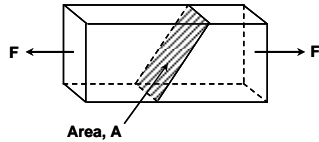
Shear Stress

Normal stress (as above):
 force per unit area carried *perpendicular* to a plane within the material.

Shear stress:
  force per unit area carried *parallel* to a plane within the material.

For uniaxial tension, consider an arbitrary plane which is inclined to the specimen axis:

F ⟵   ⟶ F

Area, A

For equilibrium, the shaded surface must carry components of force both normal and parallel to the surface:

Or as stresses:

F_{normal} σ_{normal}

⟶ F   ⟶ $\sigma_{uniaxial}$

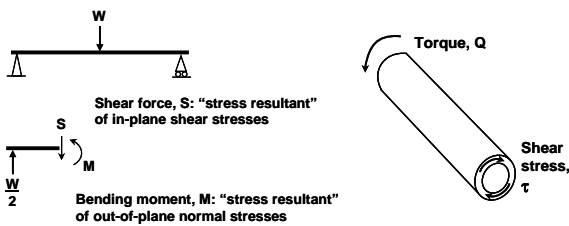F_{shear} τ_{shear}
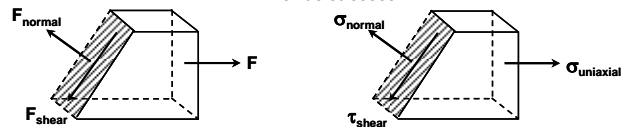
Notes:
  - usual symbol for *shear stress*: τ (Greek "tau");
  - forces may be related by resolving, but this is NOT true for the stresses
    (as stresses on the inclined plane act on a different area to the axial stress);
  - there is a simple trigonometric relationship between the stresses
    (allowing for the effect of the angle of the plane on both force components and plane area – further analysis in IB Structures).

Key points:
  - the description of a stress state depends on the orientation of the surfaces "cut" through the solid;
  - shear stresses arise in almost every loading situation, but may be off-axis from the obvious orientations chosen.

Examples of shear forces and stresses in structural components:
  - arches, beams in bending, shafts in torsion

W

Torque, Q

Shear force, S: "stress resultant" of in-plane shear stresses

S

$\frac{W}{2}$ M

Shear stress, τ

Bending moment, M: "stress resultant" of out-of-plane normal stresses

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Shear strain

Shear stresses distort the *shape* of a volume element, rather than changing its axial dimensions.

w
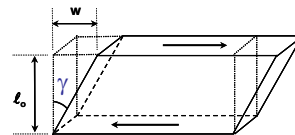
Shear strain: $\gamma = w / \ell_o$
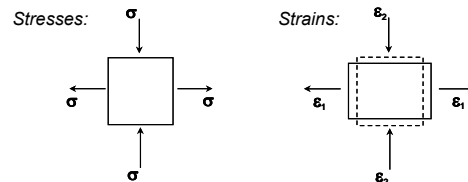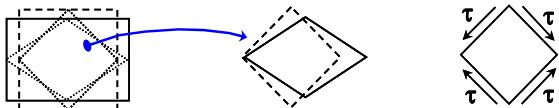
(i.e. for small strains, $\tan \gamma = \gamma$)

ℓ_o γ

As for stress, the description of the strain state depends on the axes chosen.

Consider the 2D element loaded as shown: the strains will be purely normal.

Stresses: *Strains:*

σ ε_2

σ ⟵ ⟶ σ ε_1 ⟵ ⟶ ε_1

σ ε_2

Imagine the lines joining the mid-points of the original square, before and after the deformation:

τ τ

τ τ

Pure normal strain ≡ Pure shear strain   Corresponding
(tension/   at 45°   pure shear
compression)   stress state

Simple analysis relates:
  - the pure shear stress τ to the biaxial tension/compression stress σ
  - the shear strain γ to the normal strains, ε_1 and ε_2

Shear Modulus

In some loading situations, the stiffness in shear is relevant (e.g. twisting of a shaft loaded in torsion). The *shear modulus G* is defined as:

$$G = \frac{Shear\ stress}{Shear\ strain} = \frac{\tau}{\gamma} \quad (units:\ GPa)$$

Using the normal–shear relationships between stress and strain (for pure shear) and Hooke's Law, the shear modulus G and Young's modulus E are related by:

$$G = \frac{E}{2(1 + \nu)}$$

Typical value for crystalline materials ($\nu \approx 0.33$): $G = 3/8\ E$

2.2.4 Summary: Relationships between elastic constants

(i) Four elastic constants have been defined: *E, ν, K* and *G*.

    The relationships between them are:

$$K = \frac{E}{3(1 - 2\nu)} \quad \text{and} \quad G = \frac{E}{2(1 + \nu)}$$

(ii) 3D elastic analysis assumes that the material is *isotropic*
    (i.e. same properties in all directions):
      - OK for crystalline materials and some polymers;
      - not for aligned materials: drawn polymers, wood or fibre composites.

(iii) For isotropic materials there are only *two independent elastic constants* –
    given any two, the others can be found.

2.3 Analysis of Stress and Strain

Analysis of stress and strain is fundamental to Structures, Materials and Mechanical Design.

The Part I Structures courses cover many in depth:
    - truss deflection and failure, pressure vessels,
      beam stresses and deflections, torsion of shafts

These solutions are used in Materials for *material selection* problems.

Two additional situations are addressed here:
    - *constrained deformation*
   - *thermal stress and strain*.

2.3.1 Constrained Deformation

Under uniaxial tension (or compression) elastic materials strain laterally (Poisson's ratio).

What if the material is constrained: i.e. lateral strain is prevented?

Example: constrained compression of a cube

Consider a cube of material fitted into a square-section slot in a rigid plate, and loaded with a compressive stress σ_1.
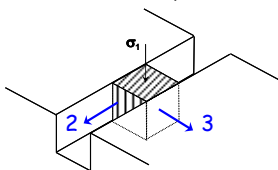
2-direction: parallel to the slot
3-direction: across the slot

Strain in the 3-direction is zero:

$$\varepsilon_3 = 0$$

The material tries to expand in the 3-direction – a *compressive stress* σ_3 is induced to prevent this. From Hooke's Law in 3D:

$$\varepsilon_3 = 0 = -\frac{\nu \,\sigma_1}{E} + \frac{\sigma_3}{E} \quad (\text{note } \sigma_2 = 0)$$

Hence: $\sigma_3 = \nu\,\sigma_1$ (both compressive)

Strain in the 1-direction (due to both stresses) is given by:

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\,\sigma_3}{E} = \frac{\sigma_1}{E} - \frac{\nu^2\,\sigma_1}{E} = \frac{\sigma_1}{E}(1 - \nu^2)$$

Hence the "effective modulus", $\dfrac{\sigma_1}{\varepsilon_1} = \dfrac{E}{(1 - \nu^2)}$ (i.e. > E)

Key points:
    - constraint induces a *transverse stress*: nominally uniaxial loads generate
      biaxial (or triaxial) stresses in the material;
    - the apparent material stiffness is increased, by a factor $1/(1 - \nu^2)$.

How much is this factor?
    - crystalline materials ($\nu \approx 0.33$): ≈ 1.12 (12% stiffer)
    - rubber ($\nu \approx 0.5$): ≈ 1.33 (33% stiffer)

The effect is even more marked in rubber if there is constraint in *both* transverse directions (since rubber is *incompressible*).

e.g. *shoe soles*:
    - thin layers of rubber loaded in compression
    - constrained (by friction) from expanding sideways
    - solid rubber soles: feel very stiff (unexpected when rubber is chosen
      for its low Young's modulus, to provide cushioning!)

How are shoe soles designed to give a springy response (Ex. Paper 2)?

2.3.2 Thermal Stress and Strain

All materials expand as temperature rises. To a good approximation, the strain caused by a temperature change, ΔT, is proportional to ΔT.

Strain/unit temperature change = *thermal expansion coefficient, α* (units: K^{-1}).

Typical values:

	Material	$\alpha \times 10^{-6}$ (K^{-1})
Ceramic	Alumina	9.0
Metals	Steels	12
	Aluminium Alloy	22
	"Invar" (Ni-Fe alloy)	1.0
Polymer	Nylon	145

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Thermal expansion is important in design when:

- *constrained expansion or contraction* (inducing *thermal stresses*);

- *temperature gradients* exist across a product, again inducing thermal stresses (e.g. heat treatment of metals);

- *dissimilar materials* are joined and then subjected to temperature change, giving *differential thermal expansion/contraction*, and induced stresses.

The last of these is exploited directly in the *bimetallic strip*:
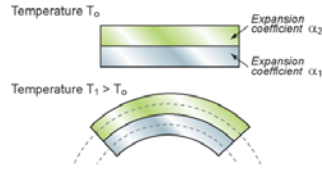
Example 1: *railway track*

Estimate the stress induced in a continuous steel railway track subjected to a 40°C temperature rise above an initial (stress-free) temperature.

Thermal strain $\varepsilon_{thermal} = \alpha \, \Delta T$

Total strain = 0, hence: $\varepsilon_{elastic} = - \varepsilon_{thermal}$

Induced elastic stress: $\sigma = - E \, \alpha \, \Delta T$

Values for steel: $\alpha = 12 \times 10^{-6} \, K^{-1}$, $E = 210$ GPa

For $\Delta T = 40$ K:

$$\sigma = -101 \text{ MPa}$$
$$(cf. \ max. \ elastic \ stress \approx 400 \text{ MPa})$$

Adelaide, January 2009

In compression this stress may cause *buckling*. Hence must leave expansion gaps, or install under tension.

Example 2: *constrained surface layers*

Many coating technologies (for *wear or corrosion resistance*) deposit a thin surface layer of a different material on a component at high temperature.

Consider a thin film on a component of thickness >> film thickness, x, and examine a length ℓ_o (at high temperature) away from the edges of the specimen:

Thermal expansion coefficients:

Surface layer, α_1

Substrate, α_2

ℓ_o

x

$\gg x$
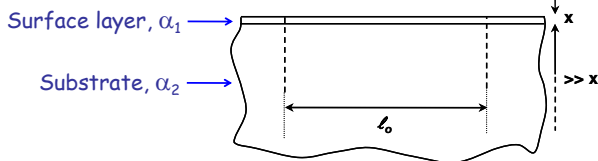
On cooling, *differential contraction* of substrate and surface layer can induce *thermal stresses*.

First imagine the layer is *not* attached to the substrate, and cool component by a temperature drop ΔT:

$\ell_o \, \alpha_1 \, \Delta T$

$\ell_o \, \alpha_2 \, \Delta T$

Thermal contractions

x

$\gg x$

ℓ_o

NB: here $\alpha_1 > \alpha_2$
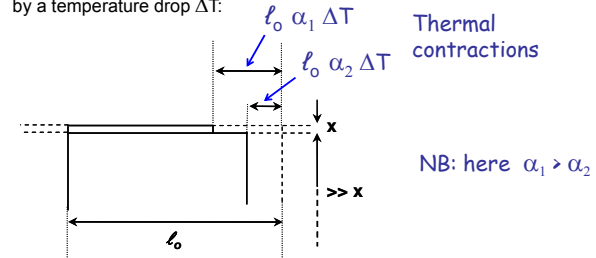
For $\alpha_1 > \alpha_2$, surface layer is restrained from contracting by the substrate and goes into *tension*, balanced by *compression* in the substrate.

As the surface layer is thin compared to the substrate:
- compressive stress negligible (large area to provide balancing force);
- final length = that of substrate = $\ell_o \, (1 - \alpha_2 \, \Delta T)$

Superpose tensile stress in surface layer, to increase its length from its contracted length, to match that of the substrate:

- change in length: $\Delta \ell = \ell_o \, \alpha_1 \, \Delta T - \ell_o \, \alpha_2 \, \Delta T$

- strain in surface layer: $\dfrac{\Delta \ell}{\ell_o \, (1 - \alpha_1 \, \Delta T)} = \dfrac{(\alpha_1 - \alpha_2) \, \Delta T}{(1 - \alpha_1 \, \Delta T)}$

Since $(\alpha_1 \, \Delta T) \ll 1$, strain = $(\alpha_1 - \alpha_2) \, \Delta T$

Notes:
- if $\alpha_1 < \alpha_2$, analysis still valid, but surface layer goes into *compression*.
- surface stress is *biaxial* (and equal in all directions): use 3D Hooke's Law to find biaxial stress in surface layer that produces this strain (Ex. Paper 2).
- key result is: stress in film $\propto (\alpha_1 - \alpha_2)$ (+ve: tension)

**2.4 Measurement of Young's Modulus**

Wide range of methods for measuring Young's modulus:
- tensile testing (Experiment 4)
- bending stiffness of a beam
- natural frequency of vibration (beams, plates – IB Experiment M1)
- speed of sound in the material, $\sqrt{E / \rho}$

(Notes below on practicalities, accuracy etc.)
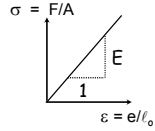

Tensile Testing:

Uniaxial tension applied to sample of standard, uniform cross-section:
 e.g. Hounsfield tensometer (Materials Lab, Experiment 4)
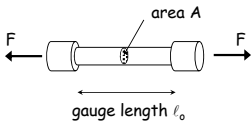
Measure extension of gauge length, from machine extension, or directly from the sample using a *strain gauge*.

Convert load vs. extension graph to stress vs. strain: gradient *E*.

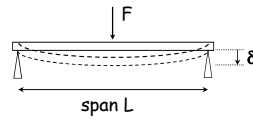$\sigma = F/A$

E

$\varepsilon = e/\ell_o$

Accuracy:
 - elastic extensions small (< 0.1%), difficult to measure precisely
 - measurement from machine must allow for flexure of machine
 - strain gauge much higher precision

Bending Stiffness of a Beam:

Beam of uniform cross-section loaded in 3-point bending:

Stiffness (F/δ) related to E via beam theory (IA Structures).

From Structures Databook: $\delta = \dfrac{F L^3}{48\, E\, I}$

EI is the *flexural rigidity*; I is the *second moment of area*, and captures the effect of cross-section size and shape:

 e.g. for a square section of side-length D: $I = D^4/12$

Hence for 3-point bend of square section beam: $E = \dfrac{(F/\delta)L^3}{4 D^4}$

Accuracy:
 - bending gives much more deflection for given load than tension
 - E sensitive to L and D: requires accurate measurement of dimensions

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Natural frequency of vibration (beams and plates)

Natural frequency of vibration in bending depends on *flexural rigidity, EI*
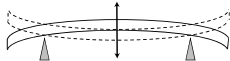
Beam supported at nodal points and set vibrating:

Plate supported on soft mounts over a loudspeaker emitting note of adjustable frequency.

"Chladni patterns" – nodal lines (revealed using tea leaves)

(Note: this method is also a good way to measure *Poisson's ratio*)

For video demo, see: http://www-materials.eng.cam.ac.uk/HRSMaterials/chladni_demo.html

Accuracy:
 - measuring frequency more accurate than deflection
 - calculated E still sensitive to beam/plate dimensions (through I and L)

Speed of Sound in the material:

The speed of sound v_t in a solid depends on stiffness of inter-atomic bonds. Young's modulus E also depends on bond stiffness (see later notes).

Hence measure E by measuring v_t:
 - strike a bar of material on one end
 - time the longitudinal wave reflected from far end of the bar

Dimensional analysis: $E = v_t^2\, \rho$, where ρ is the density

 i.e. speed of sound $v_t = \sqrt{E/\rho}$

e.g. for steel: E = 210 GPa, ρ = 7800 kg/m^3

 $v_t = (E/\rho)^{1/2} = (210\times10^9 / 7800)^{1/2} = 5188$ m/s

Accuracy:
 - for 1m long bar, time between pulses $\approx$ 1/5000 = 0.2 ms
 - accuracy depends on precise time measurement, which is relatively easy with piezoelectric transducers

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**2.5 Data for Young's Modulus – Material Property Charts**

Data sources for Young's Modulus (and other properties):
 - *Materials Databook*
 - *Cambridge Engineering Selector* (CES) databases

Material Property Charts
 - graphs with two properties as axes
 - facilitates material comparison and making trade-offs in design

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Material Property Chart: Young's Modulus vs. Density (Materials Databook)
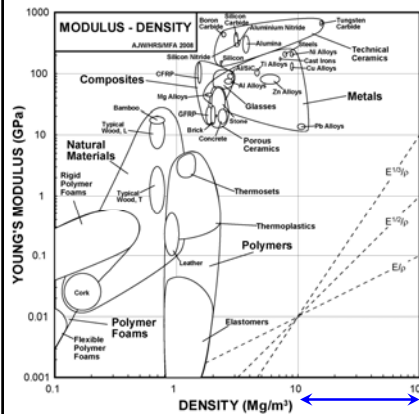
Notes on Property Charts:
- log scales: properties span many orders of magnitude
- ranges of the two properties define a "bubble" on the chart
    (usually drawn as an ellipse)
- materials cluster by class, enclosed by an envelope

Notes on Young's modulus:
- Young's modulus for solid materials spans over 6 orders of magnitude
- metals, ceramics and composites are stiff materials
- polymers, natural materials and foams are generally much less stiff,
    and cover a very wide range of stiffness

Notes on Density:
- density for solid materials spans a factor of 20
- polymer foams and porous natural materials (e.g. wood) extend the
    range to much lower values
- most ceramics, and composites, have similar densities to the
    "light metals" (Mg, Al, Ti); steels and other metals have high densities
- only porous materials and the lightest polymers float in water (ρ = 1 Mg/m^3)

Physical observations:
- materials in a given class (metals, ceramics etc) cluster together
    (indicating similarity in underlying physical basis of the properties)
- general trend for ceramics and metals: Young's modulus increases with
    density (though with exceptions, such as Pb alloys)
- individual classes of metals (e.g. steels, Ti alloys etc) have well-defined
    values of Young's modulus and density (small bubbles)
- polymers cover a wide range of Young's modulus, but their densities are
    all around 1-2 Mg/m^3; individual polymers are omitted for clarity

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3. Microstructure of Engineering Materials I

3.1 Introduction: Length-scales in engineering materials

Microstructure for Engineers
Engineering spans length scales from kilometre-scale structures to electronic
devices and micro-electro-mechanical-systems (MEMS) at fractions of 1 mm.
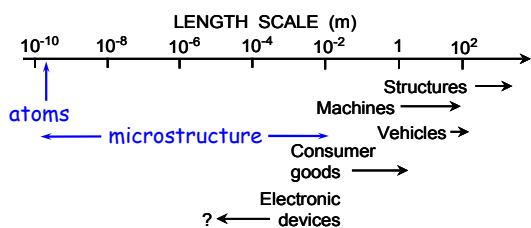
Atoms: typical radius = 0.1 – 0.2 nm
Microstructure: from atom-scale defects, compounds of atoms, "grains"
(crystal regions), to "engineering" defects such as cracks, porosity and
surface roughness (at μm – mm scale).

"Teach Yourself Microstructure"
Topics covered (and assumed in lectures):
    - primary and secondary bonding
    - crystal packing in metals and ceramics, and atomic basis of density
    - polymer microstructure

Why do engineers need to know about microstructure?
• macroscopic engineering-scale *properties* (mechanical, thermal,
    electrical…) are governed directly by the *internal microstructure;*
• different features at different *length-scales* determine each property.

Understanding microstructure enables materials engineers and scientists to:
• recognise the physical origin and inherent limits on properties,
    i.e. to target best openings for developing new or improved materials;
• know what *can* be manipulated (e.g. by changes in *composition*,
    or in *processing*) and what *cannot;*
• avoid key *failure mechanisms* in design: yield, fracture, corrosion.

3.2 Crystalline Materials: Metals, Ceramics

In *crystalline materials*, atoms pack in a regular repeating lattice structure.

Metals are sometimes used in a *pure form* (e.g. Al foil, Cu conductors) but
most commonly as *alloys* (mixed with other metal and non-metal elements):
• *Steels* and *cast irons* (Fe-C, plus Mn, Ni, Cr….)
• *Al alloys* (Al+Mg, plus Cu, Zn, Si….)
• *Cu alloys* (Cu-Sn: bronze; Cu-Zn: brass)

*Ceramics* are *compounds* of metals or silicon with non-metals (O, C or N):
• *Technical ceramics*: alumina, silicon carbide, silicon nitride
• *Glasses* (based on silica)
• *Porous ceramics*: brick, concrete, pottery

Properties and applications of common metals and alloys, and ceramics,
are tabulated in the *Materials Databook* (and in the *CES software*).

3.3 Atomic Basis of Elasticity of Metals and Ceramics

Elastic Response of Metals and Ceramics
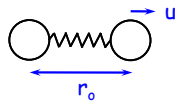Primary bonds behave as stiff elastic springs,
with force–displacement response of the form:
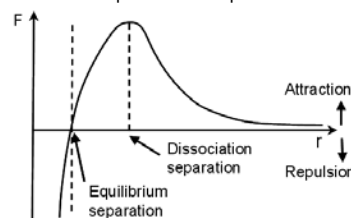
Gradient of the F– r response
(at the equilibrium spacing r_o)
is the *bond stiffness* S_o:

$$S_o = \left(\frac{dF}{dr} \right)_{r=r_o}$$

For a *small* displacement from equilibrium, *u*:

Restoring *force per atom* = $S_o u$

and the *strain* = u / r_o

Number of bonds/unit area is fixed (by the lattice geometry)
Area per bond is of order r_o^2

Hence the *stress* $\approx S_o u / r_o^2$

i.e. Young's modulus, E = stress/strain $\approx S_o / r_o$

Key points:
- Young's modulus of a component directly reflects the bond stiffness, on a length scale 10^{10} times smaller!
- atomic F–u response is linear, giving linear elasticity at the macro scale;
- primary bond stiffness and atomic packing are physically prescribed;
- *little scope for manipulating Young's modulus* via the crystal structure;
- recall that the same is true for *density* (*Teach yourself microstructure*).

Properties of Alloys
Alloys are mixtures of elements, forming *solid solutions* and *compounds*.

Density
From the hard sphere model, solid solutions and compounds will form densities between those of the pure elements.
Example densities (Mg/m^3):

pure Cu: 8.9, pure Zn: 7.1, brass (Cu-40%Zn): 8.1

Young's Modulus
Solid solutions contain a mixture of different *bond stiffnesses* (A-A, A-B, B-B):
$\Rightarrow$ Young's modulus of A-B solutions somewhere between pure A and pure B.

Compounds have *stiffer* bonds, and higher modulus – the stronger chemical bond is a major reason why the compound forms.

Example: Copper-nickel alloys form solid solutions of Cu and Ni across *all* compositions (used for conductors, electroplating, coinage).

Data for E across the composition range (from CES):

Alloy	E (GPa)
Pure Cu	122
Cu – 10% Ni	125
Cu – 30% Ni	148
Cu – 70% Ni	158
Pure Ni	190

Approximate linear "rule of mixtures"
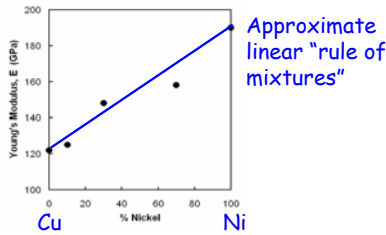
Summary: key points
- Young's modulus and density of crystalline materials are governed directly by atomic packing and atomic bonding;
- For alloys, both properties are well-defined by the composition, with no scope for change by processing.

A short aside: amorphous metals
Metals crystallise easily – some can be forced to retain an *amorphous structure* if cooled very fast.

Recently, *bulk amorphous metals* have been produced which solidify at conventional rates, using very unusual compositions:
e.g. Be–Zr–Ti–Cu–Ni–Y alloy!
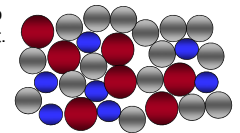
These alloys contain atoms of *widely different sizes*, making regular crystal packing difficult.

Amorphous metals have unusual properties:
- mechanically and magnetically hard
- very low damping (little energy lost in elastic collisions).

3.4 Polymer Elasticity

Classes of polymers
Thermoplastics: polyethylene (low & high density: LDPE, HDPE), polyvinylchloride (PVC), polypropylene (PP), polystyrene (PS) ….
Trade-names commonly used: nylon, acrylic, teflon, perspex…

Elastomers: rubbers, neoprene ….

← these 4 dominate

Thermosets: epoxies, phenolics, polyurethane ….

Polymer nomenclature, properties and typical applications are summarised in the Materials Databook and the CES software.

Polymer "Alloys"
Polymer molecular chemistries are well-defined, and different polymers don't mix as freely as metallic atoms.
There are two mechanisms for "alloying" polymers:
Copolymers: more than one monomer polymerised together – only a few combinations will do this, e.g. acrylonitrile, butadiene, styrene (ABS).
Polymer blends: molecular-scale mixtures of two polymer chains, without cross-linking.

Glass transition, melting and decomposition temperatures
In *crystalline materials and glasses*, the breaking of *primary* bonds by thermal energy gives a well-defined *melting point*, T_m.
In *polymers*, the weaker *secondary* bonds are overcome by thermal energy at a lower temperature: the *glass transition temperature*, T_g.

Typical values for T_g (see Materials Databook):

Polymer	T_g (°C)
Acrylic (PMMA)	85 - 165
Polystyrene (PS)	74 - 110
Polyethylene (PE)	− 25 - − 15
Natural Rubber	− 78 - − 63

Compare with melting points T_m of primary bonded crystalline materials:
- metallic bond: Al 660^oC, Fe 1534^oC
- ionic bond: Alumina Al_2O_3 2050^oC
- covalent bond: Diamond (100% covalent bonds) 3800^oC

Above T_g, the behaviour differs between the polymer classes:
- *Amorphous thermoplastics*: melt to a viscous liquid (entangled molecules slide over one another).
- *Semi-crystalline thermoplastics*: amorphous regions melt, crystalline regions survive to a higher *melting point*, T_m (typically $T_m \approx 1.5\,T_g$, in K), above which a viscous liquid forms.
- *Elastomers* and *thermosets*: secondary bonds melt at T_g but cross-links do not – on heating the polymer does not melt, but decomposes or burns.

Consequences for *processing* and *environmental impact*:

Thermoplastics: - easy to re-mould, weld, and recycle
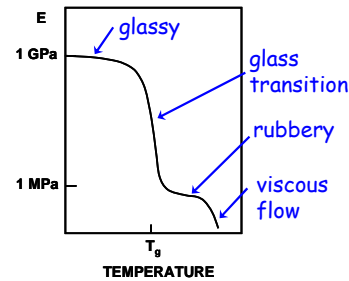   - viscosity falls with T: mould 150°C above T_g

Elastomers/Thermosets: - mould once-only
                           - cannot recycle (limited re-use)

Elastic Response of Polymers

Elasticity of polymers:
- is sensitive to *temperature* (relative to T_g) and *rate of loading*
- differs between polymer classes, due to *crystallinity* and *cross-linking*.

(1a) Amorphous thermoplastics

Glassy region ($T < T_g$):
- loading stretches low stiffness secondary bonds
- E of order 1–3 GPa
  (cf: metals, 40–200GPa; ceramics, 100–1000 GPa)

Glass transition ($T \approx T_g$):
- segments of chain melt, and modulus falls steeply
- *reptation* takes place – molecules slide past one another
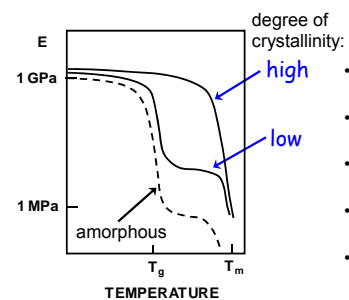- transition not abrupt (range of bond lengths in amorphous microstructure)

Rubbery region/viscous flow ($T > T_g$):
- very low stiffness above T_g (roughly 1000 times lower: a few MPa)
- rubbery elasticity is due to entanglement points
- above $1.4\,T_g$, chains all slip: viscous flow

Footnote – effect of loading rate:
- deformation around and above T_g relies on molecular sliding, which is sensitive to the rate of deformation
- hence E depends on the loading rate
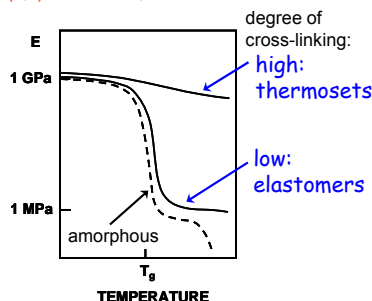- T_g must be defined at a reference loading rate

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(1b) Semi-crystalline thermoplastics: effect of crystallinity

- Crystalline regions stiffer than amorphous: higher E below T_g.
- Glass transition: no effect on crystalline regions.
- Rubbery region: E increases with degree of crystallinity
- At high crystalline fractions, glass transition has no effect on E.
- At T_m, crystalline regions melt: viscous flow.

(2,3) Elastomers, thermosets: effect of cross-linking

Elastomers:
- Below T_g (well below room temperature): glassy
- Above T_g: pronounced rubbery region.

In rubbery region:
- large recoverable strains at low stresses
- chains unravel extensively, pulled back by cross-links when the load is released.

Thermosets:
- Highly cross-linked, no effect of glass transition.
- Stiffer than thermoplastics in the glassy region; E falls slowly on heating.

Summary – key points:
- Young's modulus and density of polymers are governed directly by molecular packing and bonding;
- elastic response is sensitive to detail in molecular structure, in particular the degree of crystallinity or cross-linking (defining the polymer classes);
- significant drop in modulus in thermoplastics and elastomers at the glass transition (which itself depends on the rate of loading);
- both E and ρ are much lower than for metals and ceramics;
- Young's modulus of polymers can be manipulated more than in metals (by changing molecular weight, polymer chemistry, and degree of crystallinity or cross-linking – via processing).

3.5 Manipulating Properties I

3.5.1 Foams

Foams are *porous solids*. Porous, cellular solids are found extensively in nature – wood is a highly efficient natural engineering material.

Commercial foams use polymers of various stiffnesses, with open or closed cells. Recently, ceramic and metallic foams have been developed.

Foams are described by their *relative density* $= \dfrac{\text{density of foam}}{\text{density of solid used}}$
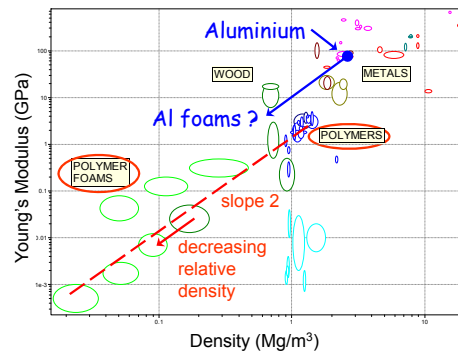
Simple model for *Young's modulus of a foam*:

- idealised unit of material as shown
- elastic response dominated by *bending* of the solid ligaments
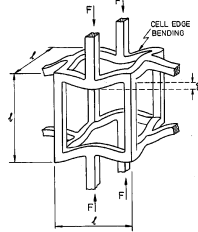
Key result is: $\dfrac{E_{foam}}{E_{solid}} = \left(\dfrac{\rho_{foam}}{\rho_{solid}} \right)^2$

On Young's modulus–density chart: lines of slope 2 lead from solid to foam.

The theory works well, enabling prediction of E, ρ of "new" materials, e.g. *aluminium foams*

#### 3.5.2 Composite materials

Composite materials combine two materials to produce *new property profiles*, exploiting separate qualities of the individual components.

*Example*: CFRP (carbon fibre reinforced polymer)

Material	E (GPa)	ρ (Mg/m^3)	E/ρ	Comment
Epoxy	2.7	1.25	2.2	Low specific stiffness
C fibres	400	1.85	216	Very high specific stiffness, but not in a usable structural form
CFRP	69-150	1.55	45-100	Fibres + epoxy provides usable form, and protects fibres from damage
Structural alloys	42-210	1.8-9.0	$\approx$ 23	CFRP superior in specific stiffness to all alloys

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Three main *composite geometries*, dependent on component architecture:
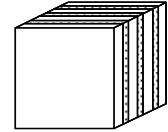
- *Particulate*: metal – ceramic: Al-SiC, WC-Co;
     polymer – ceramic: "filled polymers", e.g. glass-polyester
- *Fibres*: carbon/glass/Kevlar fibre – polymer
- *Laminates*: plywood; "GLARE", "ARALL" (Al – GFRP laminates)
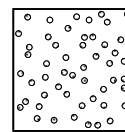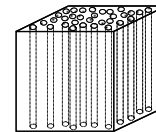
Particulate   *Fibre*   *Laminate*

Composite processing:

Particulate composites:
- add micron-scale particles to melt before casting or moulding
  (Note that many polymers are powder-filled – to provide stiffening, colour, UV resistance etc.)

Fibre composites:
- short chopped fibres: mix with resin, shape in a mould, e.g. canoes
- long continuous fibres:
  - lay out fibre mats in a mould, and infiltrate with resin
  - combine fibre+resin in "*prepreg*" plies, then laminate

Laminates:
- wood etc: stack and glue thin layers
- long-fibre composites: stack multiple layers of prepreg with different fibre orientations, hot form to shape, curing the resin

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Estimates for Composite Properties

Composites are defined by the *volume fraction* V_f of one of the components, e.g. the volume fraction of the particle or fibre reinforcement added to a matrix material.

Density

For all composites, density follows a simple *rule of mixtures* between the densities of the two components (i.e. mass conservation):

$$\rho_c = V_f \, \rho_f \; + \; \left(1 - V_f \right) \rho_m$$

Note subscript notation:
   c = composite, m = matrix, f = reinforcement (as often *fibres*)

Panel 1 (top-left)

Young's Modulus

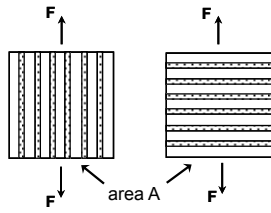Laminated composite, in edge view:

Layers have modulus E_f and E_m
  ($E_f > E_m$)

V_f = volume fraction of stiffer (E_f)
  material.

Stiffness differs parallel and
perpendicular to the layers –
the material is *anisotropic*.

F   **F**

F   area A   **F**

Loading *parallel* to the layers: <u>strain</u> same in both layers

Loading *perpendicular* to the layers: <u>stress</u> same in both layers

Panel 2 (top-right)

Young's Modulus parallel to layers   (for derivations – see *Appendix*)

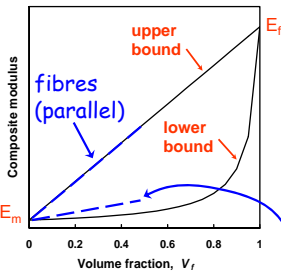*Longitudinal modulus*: $E_c = \dfrac{\sigma_c}{\varepsilon_c} = V_f\, E_f + (1 - V_f)\, E_m$

(denoted E_{II} in Databook)   *i.e. rule of mixtures for E*

Young's Modulus perpendicular to layers

Transverse modulus: $E_c = \left(\dfrac{V_f}{E_f} + \dfrac{(1 - V_f)}{E_m} \right)^{-1}$

(denoted $E_\perp$ in Databook)   *i.e. rule of mixtures for 1/E*

These "Voigt-Reuss equations" are *upper and lower bounds* for E:
  composite moduli must lie between (or on) these limits.

Panel 3 (middle-left)

Composite modulus (y-axis)
Volume fraction, V_f (x-axis): 0, 0.2, 0.4, 0.6, 0.8, 1

E_f
E_m
upper bound
lower bound
fibres (parallel)
particulate
+ fibres (perpendicular)

Particulate composites ?
 • isotropic (same in all directions)
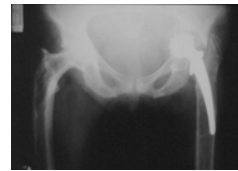 • close to lower bound (equal stress)

Fibre composites ?
 • anisotropic (stiffer parallel to fibres)
 • parallel to fibres:
   upper bound exact (equal strain)
 • perpendicular to fibres:
   close to lower bound (equal stress)

*Note*: practical upper limit on V_f in fibre and particulate composites, $\approx$ 50%.

Panel 4 (middle-right)

Case study: artificial bone for prosthetic implants

Problem: Prosthetic hip implants – Ti alloy or stainless steel (15-30 x stiffer than bone). Load transfer can be poor – the bone *resorbs* under too much stress.

Possible solution: Particulate composite – *HDPE + hydroxyapatite*, HA (the mineral found in bone, but manufactured artificially).

Images courtesy:
E. Tanner, QMC London
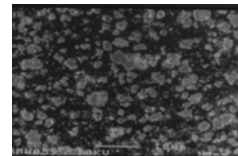
HDPE
HA

Prototype HDPE-HA composite   **Fracture surface**

 • good *biocompatibility*
 • potentially close *match with E for bone* (see Examples Paper 2)

Panel 5 (bottom-left)

Appendix: Derivation of composite Young's modulus parallel to layers

Total load F = load in *f* layers + load in *m* layers = $A_f\, \sigma_f + A_m\, \sigma_m$

where A_f, A_m = cross-sectional area of *f* and *m* layers, respectively

Average stress in composite, σ_c = F/A = $\dfrac{A_f}{A}\sigma_f + \dfrac{A_m}{A}\sigma_m$

But *area* fractions of *f* and *m* layers: $\dfrac{A_f}{A} = V_f$ and $\dfrac{A_m}{A} = (1 - V_f)$

Hence stresses follow a rule of mixtures: $\sigma_c = V_f\, \sigma_f + (1 - V_f)\, \sigma_m$

Substitute for stresses using Hooke's Law: $\sigma_f = E_f\, \varepsilon_f$, $\sigma_m = E_m\, \varepsilon_m$

  Hence: $\sigma_c = V_f\, E_f\, \varepsilon_f + (1 - V_f)\, E_m\, \varepsilon_m$

But *strains same* in composite and each material: $\varepsilon_c = \varepsilon_f = \varepsilon_m$

Longitudinal modulus: $E_c = \dfrac{\sigma_c}{\varepsilon_c} = V_f\, E_f + (1 - V_f)\, E_m$

(denoted E_{II} in Databook)

Panel 6 (bottom-right)

Derivation of composite Young's Modulus perpendicular to layers

Similar argument followed for the *transverse modulus*
 • *stress* the same in both materials
 • V_f and $(1 - V_f)$ correspond to "length fractions" in loading direction
 • *total strain* found by noting that total *extension* is the sum of the
   extensions in the two materials (i.e. *strain* follows a rule of mixtures)

Complete this derivation as an exercise (for worked solution – see online resources).

Transverse modulus: $E_c = \left(\dfrac{V_f}{E_f} + \dfrac{(1 - V_f)}{E_m} \right)^{-1}$

(denoted $E_\perp$ in Databook)