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Section 4 covers Ex. Paper 3, Q.1-3; Section 5: Q.4-8

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*Background Web resources: [www.aluminium.matter.org.uk](http://www.aluminium.matter.org.uk)*
(see: *Materials Science and Engineering: Mechanical Properties – Property Definitions*  
Introduction to Property Charts)

### 4. Stiffness-limited Component Design

#### 4.1 Property trade-offs, e.g. Stiffness at low weight or cost

Stiffness is often an important design requirement:
- buildings, vehicles, sports goods, furniture, packaging etc.

Recall that component stiffness is governed by:
- material (i.e. Young’s modulus)
- cross-sectional size + shape (tube, I-beam …)
- mode of loading (tension, bending)

Material choice is determined by:
- the additional *constraints* placed on the design
- the overall *objective*:

*Performance-driven:* Minimum

*Cost-driven:* Minimum

*Sustainability-driven:* Minimum

This implies the need to trade off material properties against one another:
- a stiffer material enables a smaller section to be used
- but will it weigh less? or cost less?

It depends on the *combination* of Young’s modulus with density (or cost/kg).

*Material property charts* are convenient for examining property trade-offs.
4.2 Selection of light, stiff materials

Property limits

A possible approach to lightweight, stiffness-limited design:
- set limits on Young’s modulus and density
  (e.g. in comparison with an existing material in an application)

Example: which materials might replace steel for lighter vehicles?

From property chart:
These are:
- brittle, i.e. fail by fast fracture (Easter Term)
- unsuitable for vehicles.

But other materials do compete with steel:
- simple limits fail to capture the trade-off between E and \( \rho \)
- for this we need a performance index
**Performance indices**

To optimise performance, a systematic analysis is required of the objectives and constraints in the design specification.

The procedure:

1. Identify **objective** (what is to be maximised or minimised, e.g. mass)
2. Identify **functional constraints** (e.g. specified stiffness?)
3. Identify **geometrical constraints** (which dimensions are fixed, and which can vary?)
4. Eliminate **free variables**, and identify **performance index** of properties.

**Example**: light, tensile tie

A lightweight tensile tie of specified length $L$ is required to carry a load $F$. The maximum allowable extension is $\delta$.

The tie has a uniform prismatic cross-section, but its area $A$ may be varied.

Step 1: **Objective**: minimum mass

Step 2: **Functional constraint**: specified stiffness

Step 3: **Geometric constraints**: fixed $L$, free variable $A$

Hence stiffness constraint becomes:

(i.e. as material is changed, area is adjusted to give required stiffness).

Step 4: **Eliminate free variable** $A$ in the objective equation:

i.e. mass is minimised by maximising the group of properties

**Key points**:

- $E/\rho$ is a **performance (or merit) index**
- $E/\rho$ is called the **specific stiffness** (though it is not always the optimum combination – see below)
- the best material can be identified even if the values of the constants in the design ($F, \delta, L$) are not given.
Selection procedure

Either: calculate \( \frac{E}{\rho} \) from tabulated \( E \) and \( \rho \) data (e.g. Materials Databook).
Or (better): graphical method, using \( E-\rho \) property chart:

Let \( \frac{E}{\rho} = \text{constant, } C \)

Taking logs:

Hence:

i.e. on log(\( E \)) vs. log(\( \rho \)) property chart:

Selection results (a short-list, not just “the best”):

- Technical Ceramics
- Composites (CFRP, Al-SiC)
- Wood (parallel to grain), Bamboo
- Metals: Mg, Al, Ti, steels, cast iron, Ni alloys:

These are the best materials on the basis of stiffness and mass alone.

Refine the selection using secondary constraints:

- simple property limits
- qualitative measures of performance and suitability (based on experience)
**Secondary constraints**

**Toughness**
- numerical criteria for toughness: covered in Easter Term
- for now, sufficient to exclude all ceramics from any structural application (particularly where impact is expected)
- same usually goes for glass (though impact-resistant glass can be manufactured, e.g. windscreens).

**Corrosion**
- materials must resist chemical attack by the service environment (e.g. water, salt water).
- corrosion is difficult to quantify, depending on subtle details (such as contact between different metals – covered in Easter Term).
- preliminary account of corrosion resistance:
  - use a simple grading of typical performance in different environments (e.g. a 5 point scale from very good to very poor)
  - see Materials Databook, and the CES software

NB: Poor resistance may not exclude a material, but implies the need for protection (e.g. painting), bringing in additional costs.

**Cost**
- product cost includes material and manufacturing cost
- as a first indicator: consider material cost only (“price/kg” bar-chart in Materials Databook)
- cost may be an objective (minimum cost) or a constraint (upper limit).

In the tensile tie example, *minimum mass* was the objective, so we set a limit on material cost (e.g. based on experience with the type of product).
- e.g. Cost/kg < £10/kg:

**Size / shape / manufacturing limits**

Not all materials may be available in the required geometry (e.g. long prismatic sections, thin-walled tubes, I-beams, complex 3D shapes)

This is usually because of *manufacturing limits*:
- **Metals and polymers**: easy to form into wide range of shapes and sizes:
  - casting/moulding (as a liquid)
  - deformation processing (as a solid or powder)
  - easy to join (e.g. by welding).
- **Ceramics**: much more limited, processed by powder compaction.
- **Fibre composites**: difficult to shape; must usually be joined by adhesives.
- **Wood**: some shape limitations;
  - well-suited to mechanical joining and adhesives.
Selection for minimum cost

Example: cheap, tensile tie

Objective now minimum material cost, rather than mass
Functional and geometric constraints as before.

Objective: material cost \( C = \)

Proceed as before, eliminating free variable A:

Cost is minimised by maximising the performance index:

Selection:
- either use tabulated data for \( E, \rho, C_m \) (time-consuming)
- or use CES to plot \( E \) vs. \( (\rho C_m) \), or a bar chart showing \( E/(\rho C_m) \)
- apply secondary constraints as before (with the exception of cost/kg, as cost is now the objective, but mass may be a limit instead)

Performance indices for light, stiff components in bending

In tension: free variable was area \( A \), but section shape did not matter
In bending: shape is used to improve the flexural rigidity of a given cross-sectional area of material, e.g. hollow tubes, I-beams etc.
Effect of shape: later in the course.
Effect of mode of loading (tension vs. bending) illustrated here via a problem with limited freedom to change the shape: lightweight panel

Example: light, stiff panel in bending

- specified span \( L \), width \( B \)
- load \( W \) in 3-point bending
- maximum allowable deflection, \( \delta \)
- rectangular cross-section, depth \( D \) may be varied
**Objective:** minimum mass

**Functional constraint:** bending stiffness \( \delta = \frac{W L^3}{48 EI} \) where \( I = \frac{BD^3}{12} \)

**Geometric constraints:** \( L, B \) fixed; \( D \) variable

Analysis on Examples Paper 3 – resulting performance index is:

On E-\( \rho \) property chart:

*Short-list of candidate materials* (ceramics excluded, due to brittleness)

- Composites (CFRP, GFRP, Al-SiC)
- Mg, Al alloys
- Wood, Bamboo, Cork
- Rigid polymer foams (Glasses)

Apply secondary constraints as before: corrosion, cost, manufacturing.
Key points:

- significant differences in competition when index changes from $E/\rho$ to $E^{1/3}/\rho$
- use of specific stiffness $E/\rho$ gives non-optimal selection for components loaded in bending
- loading details in bending (e.g. distributed load, cantilever etc) do not affect the outcome – only the constant in the stiffness equation
- best materials can again be identified without numerical values for the design constants (load, deflection, length)

Footnote:
The third guideline on the chart, $E^{1/2}/\rho$, arises for beams of constant shape (e.g. solid square sections), varying width and depth in proportion.

Summary: Performance indices on property charts
Confusion can arise in using performance indices as to what is constant and what is varying. The figure summarises the key characteristics, for $E/\rho$.

Size limits:
Size increases along a given index line (at constant mass and stiffness). In practice, maximum (or minimum) size may be limited.

In tension, $EA = \text{constant}$: upper limit on $A \Rightarrow$ lower limit on $E$

Hence a size limit corresponds to a Young’s modulus limit – a horizontal line on the chart.
(Note: fixing the position of this line does require values for the design parameters and the size limit.)

Similar arguments apply for the panel in bending (see Examples Paper 3).
4.3 Introduction to Cambridge Engineering Selector (CES) software

CES is produced by Granta Design Ltd, Cambridge, and embodies this approach to material selection (developed by Prof. M.F. Ashby in CUED).

The software provides:

- **Material property databases**
- **Material processing databases**
- **User interfaces for material selection:**
  using property limits, property charts and performance indices

The Materials Databook tables and property charts are based entirely on CES, so both may be used for problem solving.

The advantage of the software is that many design criteria can easily be imposed together, and the effect of modifying the design requirements can be evaluated automatically and interactively.

**Accessing CES:**

CES software is available free to all Engineering students and supervisors, on a time-limited licence (expires end of August).

It requires a good PC, running Windows (not Linux).

Downloadable from Web (CUED PIN protected):

http://www-h.eng.cam.ac.uk/help/engpackages.html

- CES_cut.zip: Level 1+2 databases (sufficient for Part I)
- CES.zip: Level 1,2+3 databases (larger download – use in Part II)

Download CES_cut.zip, unzip to a directory, and run setup.exe; **NB** CES requires the .NET framework software: this will install automatically, if it is not in your version of Windows already)

Do the CES Tutorial and/or follow the audio-video demos, before using CES to do the Examples Paper problems

(TEACHING WEBPAGES > FIRST YEAR INFO. > SYLLABUSES > PAPER 2 MATERIALS)

CES also available on PWF, the EIETL computer suite (restricted times), and in some colleges (library or computer room).
5. Plastic Properties of Materials

5.1 Yielding and strength in design and manufacture

As the loading on a material or component is increased, the maximum stress reaches the elastic limit. Two types of behaviour may occur:

- **Plastic deformation**: permanent strain, with the material remaining intact, until internal damage leads to a fracture;
- **Fast fracture**: cracking across the section – failure is catastrophic, with little or no inelastic deformation (next term).

**Strength-limited Design**

The maximum stress a material can carry elastically (its strength) is fundamental in elastic design:

- **safe elastic design**: find maximum working loads to avoid failure, or size/shape of section needed for a given load (e.g. structures);
- **elastic stored energy**: elastic deformation stores energy – the strength determines how much (e.g. springs, sports goods);
- **maximum elastic strain**: strength and stiffness determine the maximum elastic strain (e.g. flexible hinges with large elastic strains; differential thermal strains: may exceed elastic limit).

**Plasticity in Design and Manufacturing**

Plastic design requires consideration of loading beyond the elastic limit:

- **plastic collapse**: design of buildings includes analysis of the mechanisms of plastic collapse (IB Structures);
- **plastic energy absorption**: deforming a material dissipates energy, as exploited in protection of vehicles (e.g. automotive “crash boxes”, motorway crash barriers).

Plastic deformation is exploited in manufacturing:

- **deformation processing**: many metal shaping processes use large strain plastic deformation (e.g. forging, extrusion, rolling, machining);
- **mechanical joining methods**: e.g. clinched seams in automotive sheet.

**Plasticity in Design: other failure mechanisms**

Plastic deformation also contributes to other failure mechanisms:

- **fracture**: resistance to crack propagation requires energy dissipation – plasticity at the tip of a crack does this (next term);
- **creep**: at temperatures above 0.4 – 0.5 T_m, materials creep, i.e. deform slowly and continuously under load (next year).
5.2 Plastic response of materials

Material testing for plastic properties

Tensile testing (IA Materials laboratory)

- standard specimens loaded uniaxially in tension
- load – extension response recorded

Typical response for a ductile metal:

(1) *linear elastic* (recoverable extension)
(2) *elastic limit* – material starts to *yield*
(3) permanent *plastic strain*:
   - load increases to continue yielding
   - elastic strain recovers on unloading
(4) load reaches a maximum
(5) *necking*: deformation *localises*, specimen fails

Initial sample:  

(2) – (4):  

(4) – (5):  

Failure:  

As plastic deformation progresses:
- specimen elongates uniformly
- its cross-sectional area reduces

Volume conservation

Yielding takes place at *constant volume*, for reasons explained later.
Origin of necking

In region (2 – 4) of the load-extension curve:
• yield load increasing and area decreasing ⇒ actual true stress \( \frac{F}{A} \) to yield the material is increasing
• increase in resistance to yielding is called work hardening (microstructural origin discussed later)
• this stabilises yielding, so the whole cross-section reduces uniformly

At maximum load, \( \frac{dF}{du} = 0 \) (point 4):
• the material’s ability to work harden in tension has reduced
• at any local narrowing of the section the true stress is higher, but work hardening is now insufficient to compensate
• the plastic strain localises in this section: necking
• as the load falls (4 – 5), the sample unloads elastically, but intense strain in the neck leads to internal damage (voids) and failure

Nominal Stress and Strain

The instantaneous true stress is \( \frac{F}{A} \). For convenience the load is converted to a nominal stress using the initial area \( A_0 \):

Similarly, the extension is converted to strain using the initial gauge length \( \ell_0 \), giving a nominal strain:

Nominal stress – strain curve is therefore identical in shape to the load – extension curve, and defines a number of quantities relevant to design:
Definitions

Yield stress, $\sigma_y$: nominal stress at elastic limit, when yielding starts

Tensile strength, $\sigma_{ts}$: maximum nominal stress in tensile test

Yield strain, $\varepsilon_y$: $\sigma_y/E$

Elongation (or ductility), $\varepsilon_f$: permanent strain after failure
  (i.e. excluding elastic contribution)

Max. elastic stored energy per unit volume: area under linear-elastic region.
  Stored energy = work done in elastic deformation = $\frac{1}{2} Fu$.
  So stored elastic energy per unit volume

Maximum elastic stored energy per unit volume when $\sigma_n = \sigma_y$:

Plastic work per unit volume: area under stress-strain curve (excluding final elastic contribution), i.e. area under force-extension curve divided by initial volume $A_o \ell_o$:
  Plastic work/unit volume =

Notes:

Yield stress is a material property (does not depend on specimen dimensions)

Tensile strength is mainly useful in comparison with the yield stress – as an indication of the amount of work hardening after initial yield.

Elongation:
  • includes both the uniform plastic extension up to maximum stress, and the strain during localisation prior to failure
  • the latter strain varies with the size of specimen, so elongation is not a material property
  • it is still commonly cited as a measure of ductility (e.g. included in CES)
  • must be treated with caution – acceptable for a first comparison of materials (provided a standard test has been used)
Tensile stress–strain curve variants

Some materials show variants from the standard form illustrated above:

**Elastic–brittle:**
No plasticity, ductility = 0

**Upper and lower yield point:**
Short plateau of plastic strain before hardening sets in.

**No distinct yield point:**
Work hardening gives non-linear response from low stresses.
The nominal stress at a small plastic strain (e.g. 0.1%) is used to define yield – this is called a 0.1% proof stress, $\sigma_{0.1}$.

Nominal stress–strain curves for a range of common alloys (from Materials Databook):

Note on terminology:
- **Annealed:** softened, by heat treatment
- **Drawn:** previously hardened, by stretching

Stress–strain curves are significant for manufacturing, as well as design: materials with large tensile ductility (e.g. mild steel, annealed Cu, Al) are easier to form in tension (e.g. wire drawing, deep drawing of cans)
Compression Testing

In some circumstances it is preferable to use uniaxial compression:

- **Ceramics**: tend to fracture in tension (from internal flaws); much stronger in compression, and may yield (particularly when hot)
- **Metals**: if unavailable in suitable geometry for tension.

![Diagram of Compression Testing](image)

**Nominal stress**:

**Nominal strain**:

Area increases during test – load for yielding *rises* (due to larger area, and due to work hardening). Localisation and necking do not occur – the test geometry is stable.

**Constant volume**

\[ A_o h_o = A h \]

**Hardness**

Tension and compression tests are destructive – samples are machined to size and tested to destruction.

It is often desirable to use non-destructive testing (NDT):

- may need to return component to service after testing
- size/shape may not allow manufacture of tensile or compressive sample
- tensile/compression testing can be expensive and time-consuming

**Hardness test** – standard low-cost NDT technique for measuring strength:

*e.g.* **Vickers hardness test**

![Diagram of Vickers Hardness Test](image)

- diamond pyramid is pressed into the surface under a constant load;
- local plastic deformation occurs until load is supported;
- load removed and the resulting indent size, \( d \), is measured;
- \( A = \frac{(d/\sqrt{2})^2}{2} = \frac{d^2}{2} \)

**Hardness** defined as:

*e.g.* Vickers hardness, “HV₅” (i.e. load, \( F = 5 \) kg): conversion from \( d \) to HV tabulated for standard loads.

**Units**: usually kg/mm²
(can also be expressed in MPa: multiply by \( g \), acceleration due to gravity).
Relationship between hardness and yield stress
Under the indenter, a steep strain gradient exists – the deformation and work hardening is greatest near the indenter. Plastic analysis indicates that:
• average strain is of the order of 8%, so the hardness relates to the yield stress at this strain
• hardness (in MPa) is approximately equal to 3 \times \text{yield stress}, \ H \approx 3\sigma_y

e.g. low carbon steel:

True Stress and Strain
Nominal stress and strain are convenient for material testing, but do not take account of changes in geometry. In some circumstances it is preferable to account for geometry correctly, and to use true stress and true strain:

True Stress
For uniaxial load, true stress is: \ \sigma_t = \frac{F}{A} \ and \ nominal \ stress \ is: \ \sigma_n = \frac{F}{A_o}

For constant volume (i.e. for plastic deformation): \ A_o \ell_o = A \ell

Hence:

Nominal strain: \ \varepsilon_n = \frac{\ell - \ell_o}{\ell_o} = \frac{\ell}{\ell_o} - 1, \ i.e. \ \frac{\ell}{\ell_o} = 1 + \varepsilon_n

Hence: \ i.e. \ true \ stress > nominal \ stress \ (for \ \varepsilon_n > 0).

True stress is easily calculated from nominal stress–strain data, but only as far as the onset of necking in tension (deformation no longer uniform) – the continuation of the true stress–strain curve must be found in compression.

True Strain
True strain is more difficult to visualise – each increment of extension \ d\ell \ is divided by the current length (not the initial value), to give an increment of true strain: \ d\varepsilon = \frac{d\ell}{\ell}

Total true strain is found by integrating between original and final lengths:

\varepsilon_t = \int_{\ell_o}^{\ell} \frac{d\ell}{\ell} = \ln \left( \frac{\ell}{\ell_o} \right)

Since: \ \left( \frac{\ell}{\ell_o} \right) = (1 + \varepsilon_n), \ true \ strain \ is:

i.e. \ true \ strain < nominal \ strain \ (for \ \varepsilon_n > 0), \ and \ easily \ calculated \ from \ \varepsilon_n.
Comparative stress-strain curves (up to maximum nominal stress):

When is nominal $\approx$ true?
\[ \varepsilon_t = \ln(1 + \varepsilon_n) = \varepsilon_n + \text{higher order terms} \]

e.g. for yield strain, $\varepsilon_y$: $\varepsilon_n \approx 0.1\%$

For necking strain, $\varepsilon_{\text{neck}}$: $\varepsilon_n \approx 20\%$ (e.g. mild steel)

Tension vs. Compression
True stress and strain take account of geometry change, so true $\sigma$–$\varepsilon$ curves for tension and compression are identical.
At yield strain $\varepsilon_n \approx \varepsilon_t$ and $\sigma_n \approx \sigma_t$, hence yield stress will be the same in tension and compression.

Additivity of true strains
Some manufacturing processes apply sequential plastic strains – e.g. multi-pass rolling. Each deformation can be described by nominal or true strain.
The advantage of true strain is that it is additive: separate strains can be summed. This is not so for nominal strain (see Examples Paper 3).

5.3 Data for Strength – Material Property Charts
Strength is defined as the nominal stress at the elastic limit.
Strength data are more complex, due to the different ways in which material classes exceed their elastic limit:

- **Brittle materials** (e.g. ceramics) (tension): fracture occurs, giving a tensile strength (variable, due to variability in internal flaws)
- **Brittle materials** (compression): crushing occurs, giving a compressive strength (typically 10-15 x higher than tensile strength)
- **Ductile materials** (e.g. metals) (tension or compression): yield stress (or proof stress)
Material property charts including Strength

(1) Strength vs. Density

NB: Envelope round technical ceramics, glasses and porous ceramics shown dotted: data for compression only

Performance indices – return to these later.

Notes:
• Strength for solid materials spans over 3 orders of magnitude – natural materials and polymer foams extend the range to much lower strengths;
• Metals, ceramics and composites are the strongest materials, but there is some overlap with polymers;
• Individual classes of metals (e.g. steels, Ti alloys etc) cover much wider ranges of strength than is observed for Young’s modulus or density.

(2) Young’s Modulus vs. Strength

NB. Wide ranges in strength but not in Young’s modulus – strength is a property that can be manipulated widely by composition and processing.

Further performance indices – e.g. yield strain, elastic stored energy per unit volume

Materials Databook also includes a property chart for Fracture Toughness vs. Strength – discussed in the Easter Term lectures on fracture.