Engineering Tripos Part IA   **First Year**

Paper 2 – MATERIALS

HANDOUT 3

4. **Stiffness-limited Component Design**
   4.1 Property trade-offs: e.g. Stiffness at low weight or cost
   4.2 Selection of light, stiff materials
   4.3 Introduction to Cambridge Engineering Selector (CES) software

5. **Plastic Deformation and Properties**
   5.1 Yielding and strength in design and manufacture
   5.2 Plastic response of materials
   5.3 Data for Strength – Material Property Chart

Section 4 covers Ex. Paper 3, Q.1-3; Section 5: Q.4-8

H.R. Shercliff
hrs@eng.cam.ac.uk   February 2014

Background Web resources: www.aluminium.matter.org.uk
      (see: *Materials Science and Engineering*:
            Mechanical Properties – Property Definitions
            Introduction to Property Charts)

---

4. **Stiffness-limited Component Design**

4.1 **Property trade-offs, e.g. Stiffness at low weight or cost**

Stiffness is often an important design requirement:
   - buildings, vehicles, sports goods, furniture, packaging etc.

Recall that component stiffness is governed by:
- material (i.e. Young's modulus)
- cross-sectional size + shape (tube, I-beam …)
- mode of loading (tension, bending)

Material choice is determined by:
   - the additional *constraints* placed on the design
   - the overall *objective*:

Performance-driven: Minimum mass

Cost-driven: Minimum cost

Sustainability-driven: Minimum environmental impact
                    (e.g. life cycle energy consumption, CO_2)

NB: for transport: Minimum env. impact = minimum mass

This implies the need to trade off material properties against one another:
- a stiffer material enables a smaller section to be used
- but will it weigh less?   or cost less?

It depends on the *combination* of Young's modulus with density (or cost/kg).
Material property charts are convenient for examining property trade-offs.

4.2 **Selection of light, stiff materials**

Property limits

A possible approach to lightweight, stiffness-limited design:
- set limits on Young's modulus and density
      (e.g. in comparison with an existing material in an application)

Example: which materials might replace steel for lighter vehicles?

Stiffer and lighter than steel: E > 210 GPa, ρ < 7.6 Mg/m^3

From property chart: various ceramics

These are:
- *brittle*, i.e. fail by *fast fracture* (Easter Term)
- unsuitable for vehicles.

But other materials *do* compete with steel:
- simple limits fail to capture the *trade-off* between E and ρ
- for this we need a *performance index*

Performance indices

To optimise performance, a systematic analysis is required of the *objectives* and *constraints* in the design specification.

The procedure:

   (1) Identify *objective* (what is to be maximised or minimised, e.g. mass)

   (2) Identify *functional constraints* (e.g. specified stiffness?)

   (3) Identify *geometrical constraints* (which dimensions are fixed, and which can vary?)

   (4) Eliminate *free variables*, and identify *performance index* of properties.
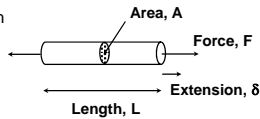
1

Example: light, tensile tie   **e.g. bicycle spoke**

A lightweight tensile tie of specified length *L* is required to carry a load *F*.

The maximum allowable extension is δ.

The tie has a uniform prismatic cross-section, but its area *A* may be varied.

Area, A
Force, F
Extension, δ
Length, L

Step 1: *Objective*: minimum *mass* $m = \rho\, L\, A$

Step 2: *Functional constraint*: specified *stiffness* $= AE/L$ (i.e. $E = \sigma/\varepsilon$)

Step 3: *Geometric constraints*: fixed *L*, free variable *A*

Hence stiffness constraint becomes: $(F/\delta)\, L = AE = $ constant

(i.e. as material is changed, area is adjusted to give required stiffness).

Step 4: *Eliminate free variable A* in the objective equation:

$$\text{Mass } m = \rho\, L\, A = L^2\,(F/\delta)\,(\rho/E), \quad \text{i.e. mass} \propto (\rho/E)$$

i.e. mass is minimised by maximising the group of properties E/ρ

Key points:
- E/ρ is a *performance (or merit) index*
- E/ρ is called the *specific stiffness* (though it is not always the optimum combination – see below)
- the best material can be identified even if the values of the constants in the design (F, δ, L) are not given.

Selection procedure

Either: calculate E/ρ from tabulated *E* and ρ data (e.g. Materials Databook).
Or (better): graphical method, using E-ρ property chart:

Let E/ρ = constant, C

Taking logs: $\log(E) - \log(\rho) = \log(C)$

Hence: $\log(E) = \log(\rho) + \log(C)$

i.e. on log(E) vs. log(ρ) property chart: **lines of slope 1**
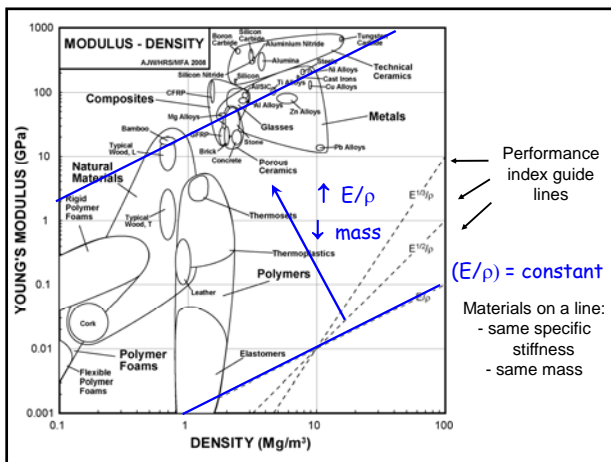
Selection results (a short-list, not just "the best"):

Technical Ceramics
Composites (CFRP, Al-SiC)
Wood (parallel to grain), Bamboo
Metals: Mg, Al, Ti, steels, cast iron, Ni alloys: **virtually same E/ρ**

These are the best materials on the basis of *stiffness and mass alone*.

Refine the selection using secondary constraints:
- simple *property limits*
- qualitative measures of *performance* and *suitability* (based on experience)

Secondary constraints
Toughness
- numerical criteria for toughness: covered in Easter Term
- for now, sufficient to exclude all ceramics from any structural application (particularly where *impact* is expected)
- same usually goes for glass (though impact-resistant glass can be manufactured, e.g. windscreens).

exclude ceramics (and glass)

Corrosion
- materials must resist chemical attack by the service environment (e.g. water, salt water).
- corrosion is difficult to quantify, depending on subtle details (such as contact between different metals – covered in Easter Term).
- preliminary account of corrosion resistance:
  - use a simple grading of typical performance in different environments (e.g. a 5 point scale from very good to very poor)
  - see *Materials Databook*, and the *CES software*

e.g. Fe and Mg alloys, wood: average or poor in salt water

NB: Poor resistance may not exclude a material, but implies the need for *protection* (e.g. painting), bringing in additional costs.

Cost
- product cost includes material and manufacturing cost
- as a first indicator: consider *material cost only* ("price/kg" bar-chart in Materials Databook)
- cost may be an *objective* (minimum cost) or a *constraint* (upper limit).

In the tensile tie example, *minimum mass* was the objective, so we set a limit on material cost (e.g. based on experience with the type of product).

e.g. Cost/kg < £10/kg: **excludes CFRP, Ti alloys**

Size / shape / manufacturing limits

Not all materials may be available in the required geometry
(e.g. long prismatic sections, thin-walled tubes, I-beams, complex 3D shapes)

This is usually because of *manufacturing limits*:

- *Metals and polymers*: easy to form into wide range of shapes and sizes:
  - casting/moulding (as a liquid)
  - deformation processing (as a solid or powder)
  - easy to join (e.g. by welding).
- *Ceramics*: much more limited, processed by powder compaction.
- *Fibre composites*: difficult to shape; must usually be joined by adhesives.
- *Wood*: some shape limitations;
  well-suited to mechanical joining and adhesives.

Selection for minimum cost

Example: cheap, tensile tie   e.g. cheap bike spokes

Objective now minimum material cost, rather than mass

Functional and geometric constraints as before.

Objective: material cost C = $m\,C_m$ = $\rho\,L\,A\,C_m$

C_m = material cost/kg

Proceed as before, eliminating free variable A:

Cost C = $L^2\,(F/\delta)\,(\rho\,C_m/E)$

i.e. Cost C $\propto$ $(\rho\,C_m/E)$

Cost is minimised by maximising the performance index: $E/(\rho\,C_m)$

Selection:
- either use tabulated data for E, ρ, C_m (time-consuming)
- or use CES to plot E vs. $(\rho\,C_m)$, or a bar chart showing $E/(\rho\,C_m)$
- apply secondary constraints as before (with the exception of cost/kg,
  as cost is now the objective, but mass may be a limit instead)

Performance indices for light, stiff components in bending

In *tension*: free variable was area A, but *section shape* did not matter

In *bending*: shape is used to improve the *flexural rigidity* of a given cross-sectional area of material, e.g. hollow tubes, I-beams etc.

Effect of *shape* : later in the course.

Effect of *mode of loading* (tension vs. bending) illustrated here via a problem with limited freedom to change the shape: *lightweight panel*

Example: light, stiff panel in bending   e.g. furniture, open staircase

- specified span L, width B
- load *W* in 3-point bending
- maximum allowable deflection, δ
- rectangular cross-section, depth D may be varied

Load, W
Width, B
Depth, D
Length, L
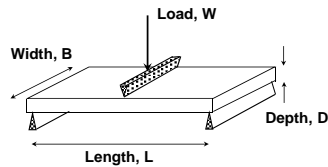
Objective: minimum mass

Functional constraint: bending stiffness $\delta = \dfrac{W\,L^3}{48\,E\,I}$ where $I = \dfrac{B\,D^3}{12}$

Geometric constraints: *L, B* fixed; *D* variable

Analysis on Examples Paper 3 – resulting performance index is: $E^{1/3}/\rho$

On E-ρ property chart: $E^{1/3}/\rho$ = constant : lines of slope 3

$(E^{1/3}/\rho)$ = constant

↑ $E^{1/3}/\rho$
↓ mass
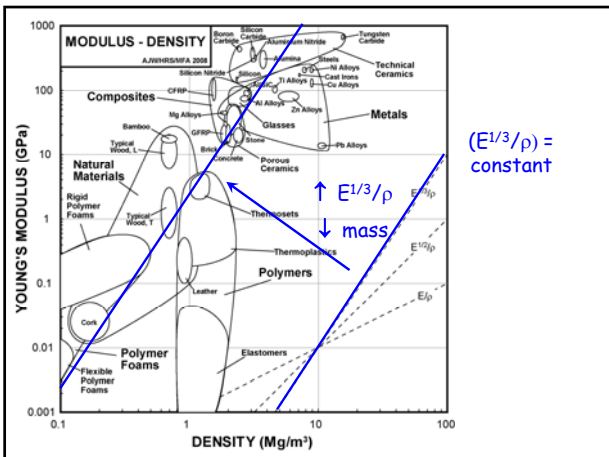
Short-list of candidate materials (ceramics excluded, due to brittleness)

Composites (CFRP, GFRP, Al-SiC)   Mg, Al alloys

Wood, Bamboo, Cork   Rigid polymer foams

(Glasses)

Apply secondary constraints as before: corrosion, cost, manufacturing.

Key points:
- significant differences in competition when index changes from E/ρ to $E^{1/3}/\rho$
- use of specific stiffness E/ρ gives *non-optimal selection* for components loaded in *bending*
- loading details in bending (e.g. distributed load, cantilever etc) do not affect the outcome – only the constant in the stiffness equation
- best materials can again be identified without numerical values for the design constants (load, deflection, length)

Footnote:

The third guideline on the chart, $E^{1/2}/\rho$, arises for beams of *constant shape* (e.g. solid square sections), varying width *and* depth in proportion.

Summary: Performance indices on property charts

Confusion can arise in using performance indices as to what is constant and what is varying. The figure summarises the key characteristics, for E/ρ.
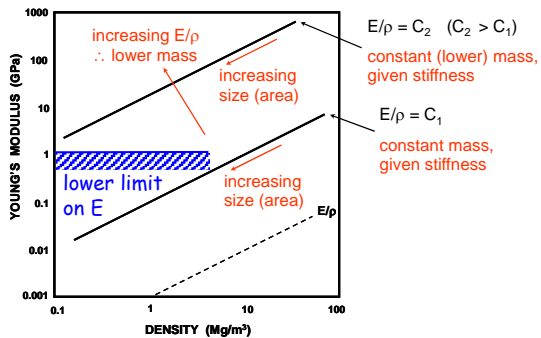
Size limits:

Size increases along a given index line (at constant mass *and* stiffness). In practice, maximum (or minimum) size may be limited.

In tension, EA = constant: upper limit on A $\Rightarrow$ lower limit on E

Hence a *size limit* corresponds to a *Young's modulus limit* – a horizontal line on the chart.

(Note: fixing the position of this line *does* require values for the design parameters and the size limit.)

Similar arguments apply for the panel in bending (see Examples Paper 3).

4.3 Introduction to Cambridge Engineering Selector (CES) software

CES is produced by Granta Design Ltd, Cambridge, and embodies this approach to material selection (developed by Prof. M.F. Ashby in CUED).

The software provides:

- *Material property databases*
- *Material processing databases* ⎤ a comprehensive data source for all years of the Tripos

- *User interfaces for material selection*:
  using property limits, property charts and performance indices

The Materials Databook tables and property charts are based entirely on CES, so both may be used for problem solving.

The advantage of the software is that many design criteria can easily be imposed together, and the effect of modifying the design requirements can be evaluated automatically and interactively.

Accessing CES:

CES software is available free to all Engineering students and supervisors, on a time-limited licence (expires end of August).

It requires a good PC, running Windows (not Linux).

Downloadable from Web (CUED PIN protected):

http://www-h.eng.cam.ac.uk/help/engpackages.html

CES_cut.zip: Level 1+2 databases (sufficient for Part I)
CES.zip: Level 1,2+3 databases (larger download – use in Part II)

Download CES_cut.zip, unzip to a directory, and run *setup.exe*;
NB CES requires the .NET framework software: this will install automatically, if it is not in your version of Windows already)

Do the *CES Tutorial* and/or follow the *audio-video demos,* before using CES to do the Examples Paper problems
(Teaching Webpages > First Year Info. > Syllabuses > Paper 2 Materials)

CES also available on PWF, the EIETL computer suite (restricted times), and in some colleges (library or computer room).

5. **Plastic Properties of Materials**
   5.1 Yielding and strength in design and manufacture
   5.2 Plastic response of materials
   5.3 Data for Strength – Material Property Chart
   Section 5 covers Examples Paper 3, Q.4-8

Background Web resources: www.aluminium.matter.org.uk
   (see: *Materials Science and Engineering*:
        Mechanical Properties – Property Definitions
        Introduction to Property Charts)

---

5. **Plastic Properties of Materials**

5.1 Yielding and strength in design and manufacture

As the loading on a material or component is increased, the maximum stress reaches the *elastic limit*. Two types of behaviour may occur:
- *Plastic deformation*: permanent strain, with the material remaining intact, until internal damage leads to a fracture;
- *Fast fracture*: cracking across the section – failure is catastrophic, with little or no inelastic deformation (next term).

Strength-limited Design

The maximum stress a material can carry elastically (its *strength*) is fundamental in *elastic design*:
- *safe elastic design*: find maximum working loads to avoid failure, or size/shape of section needed for a given load (e.g. structures);
- *elastic stored energy*: elastic deformation stores energy – the strength determines how much (e.g. springs, sports goods);
- *maximum elastic strain*: strength and stiffness determine the maximum elastic strain (e.g. flexible hinges with large elastic strains; differential thermal strains: may exceed elastic limit).

Plasticity in Design and Manufacturing

Plastic design requires consideration of loading *beyond* the elastic limit:
- *plastic collapse*: design of buildings includes analysis of the mechanisms of plastic collapse (IB Structures);
- *plastic energy absorption*: deforming a material dissipates energy, as exploited in protection of vehicles (e.g. automotive "crash boxes", motorway crash barriers).

Plastic deformation is exploited in *manufacturing*:
- *deformation processing*: many metal shaping processes use large strain plastic deformation (e.g. forging, extrusion, rolling, machining);
- *mechanical joining methods*: e.g. clinched seams in automotive sheet.

Plasticity in Design: other failure mechanisms

Plastic deformation also contributes to other *failure mechanisms*:
- *fracture*: resistance to crack propagation requires energy dissipation – plasticity at the tip of a crack does this (next term);
- *creep*: at temperatures above $0.4 - 0.5\ T_m$, materials creep, i.e. deform slowly and continuously under load (next year).
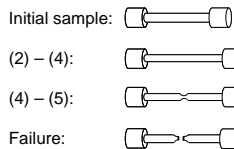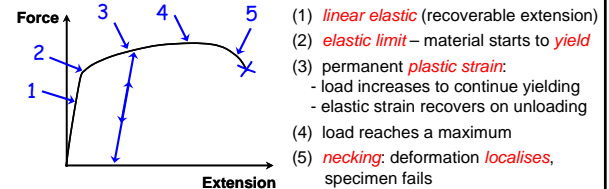

5.2 Plastic response of materials

Material testing for plastic properties

Tensile testing (IA Materials laboratory)
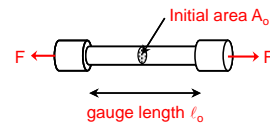
- standard specimens loaded uniaxially in tension
- load – extension response recorded

Typical response for a *ductile metal*:

(1) *linear elastic* (recoverable extension)
(2) *elastic limit* – material starts to *yield*
(3) permanent *plastic strain*:
   - load increases to continue yielding
   - elastic strain recovers on unloading
(4) load reaches a maximum
(5) *necking*: deformation *localises*, specimen fails

Initial sample:

(2) – (4):

(4) – (5): Necking

Failure:

As plastic deformation progresses:
- specimen elongates uniformly
- its cross-sectional area reduces

Volume conservation

Yielding takes place at *constant volume*, for reasons explained later.

Volume = $A_o\ \ell_o = A\ \ell$
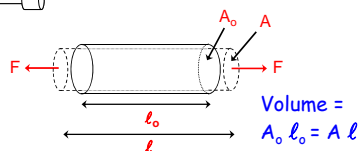
Origin of necking

In region (2 – 4) of the load-extension curve:
- yield load increasing *and* area decreasing $\Rightarrow$ actual *true stress* (F/A) to yield the material is increasing
- increase in resistance to yielding is called *work hardening* (microstructural origin discussed later)
- this stabilises yielding, so the whole cross-section reduces uniformly

At maximum load, , $dF/du = 0$ (point 4):
- the material's ability to work harden in tension has reduced
- at any local narrowing of the section the true stress is higher, but work hardening is now insufficient to compensate
- the plastic strain localises in this section: *necking*
- as the load falls (4 – 5), the sample unloads elastically, but intense strain in the neck leads to internal damage (voids) and failure

Nominal Stress and Strain

The instantaneous *true stress* is F/A. For convenience the load is converted to a *nominal stress* using the *initial area A_o*: $\sigma_n = F/A_o$

Similarly, the extension is converted to strain using the *initial gauge length ℓ_o*, giving a *nominal strain*: $\varepsilon_n = (\ell - \ell_o)/\ell_o$

Nominal stress – strain curve is therefore identical in shape to the load – extension curve, and defines a number of quantities relevant to design:
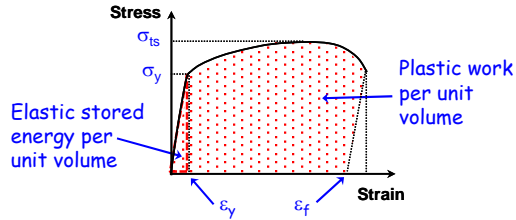
Definitions

Yield stress, σ_y: nominal stress at elastic limit, when yielding starts

Tensile strength, σ_{ts}: maximum nominal stress in tensile test

Yield strain, ε_y: σ_y/E

Elongation (or ductility), ε_f: permanent strain *after* failure
(i.e. excluding elastic contribution)

Max. elastic stored energy per unit volume: area under linear-elastic region.

Stored energy = work done in elastic deformation = ½ Fu.

So stored elastic energy per unit volume = $\frac{1}{2} Fu / (A_o\ell_o)$

$$= \tfrac{1}{2} (F/A_o)\times(u/\ell_o) = \tfrac{1}{2} \sigma_n \varepsilon_n = \tfrac{1}{2} \sigma_n^2/E$$

Maximum elastic stored energy per unit volume when $\sigma_n = \sigma_y$: $\sigma_y^2/2E$

Plastic work per unit volume: area under stress-strain curve (excluding final elastic contribution), i.e. area under force-extension curve divided by initial volume $A_o\ell_o$:

Plastic work/unit volume = $\displaystyle\int F\, du \,/(A_o\ell_o) = \int \sigma_n\, d\varepsilon_n$

Notes:

Yield stress is a *material property* (does not depend on specimen dimensions)

Tensile strength is mainly useful *in comparison with* the yield stress – as an indication of the amount of *work hardening* after initial yield.

Elongation:
• includes both the uniform plastic extension up to maximum stress, and the strain during localisation prior to failure
• the latter strain varies with the size of specimen, so elongation is <u>not a material property</u>
• it is still commonly cited as a measure of ductility (e.g. included in CES)
• must be treated with caution – acceptable for a first comparison of materials (provided a standard test has been used)
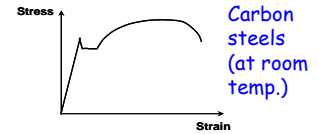
Tensile stress–strain curve variants

Some materials show variants from the standard form illustrated above:

Elastic–brittle:
No plasticity, ductility = 0

Ceramics

Carbon steels (BCC) (at low temp.)
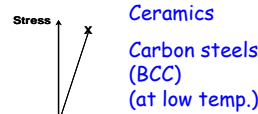
Upper and lower yield point:
Short plateau of plastic strain before hardening sets in.
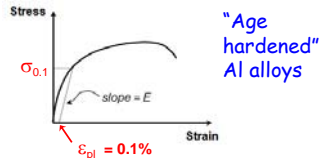

Carbon steels (at room temp.)

No distinct yield point:
Work hardening gives non-linear response from low stresses.

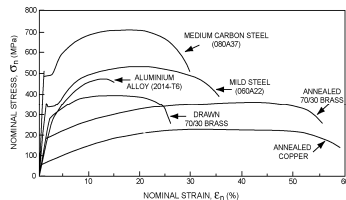The nominal stress at a small plastic strain (e.g. 0.1%) is used to define yield – this is called a *0.1% proof stress, $\sigma_{0.1}$*.

$\sigma_{0.1}$ slope = E ε_{pl} = 0.1%

"Age hardened" Al alloys

Nominal stress–strain curves

for a range of common alloys (from *Materials Databook*):

Note on terminology :
• *Annealed*: softened, by heat treatment
• *Drawn*: previously hardened, by stretching

Stress–strain curves are significant for *manufacturing*, as well as design: materials with large tensile *ductility* (e.g. mild steel, annealed Cu, Al) are easier to *form in tension* (e.g. wire drawing, deep drawing of cans)

Compression Testing

In some circumstances it is preferable to use *uniaxial compression*:
• *Ceramics*: tend to fracture in tension (from internal flaws); much stronger in compression, and may yield (particularly when hot)
• *Metals*: if unavailable in suitable geometry for tension.
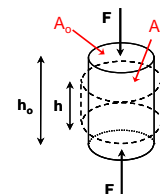
Nominal stress: $\sigma_n = -F/A_o$

Nominal strain: $\varepsilon_n = (h - h_o)/h_o$
(i.e. negative)

Area increases during test – load for yielding *rises* (due to larger area, and due to work hardening).

Localisation and necking do not occur – the test geometry is stable.

Constant volume
= $A_o h_o = A h$

Hardness

Tension and compression tests are destructive – samples are machined to size and tested to destruction.

It is often desirable to use *non-destructive testing* (NDT):

- may need to return component to service after testing
- size/shape may not allow manufacture of tensile or compressive sample
- tensile/compression testing can be expensive and time-consuming

---

Hardness test – standard low-cost NDT technique for measuring *strength*:

e.g. Vickers hardness test

F

Projected area A

d
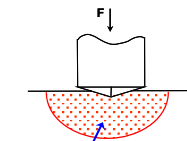
Local plastic zone

- diamond pyramid is pressed into the surface under a constant load;
- local plastic deformation occurs until load is supported;
- load removed and the resulting indent size, d, is measured;
- $A = (d/\sqrt{2})^2 = d^2/2$

Hardness defined as: $H = \dfrac{load}{projected\ area\ of\ indent} = \dfrac{F}{A}$

e.g. Vickers hardness, "HV_5" (i.e. load, F = 5 kg): conversion from d to HV tabulated for standard loads.

Units: usually kg/mm^2
(can also be expressed in MPa: multiply by g, acceleration due to gravity).

Relationship between hardness and yield stress

Under the indenter, a steep strain gradient exists – the deformation and work hardening is greatest near the indenter. Plastic analysis indicates that:

- *average strain* is of the order of 8%, so the hardness relates to the yield stress at this strain
- hardness (in MPa) is approximately equal to 3 × yield stress, $H \approx 3\sigma_y$

e.g. low carbon steel:

$HV = 120\ kg/mm^2$, $H \approx 1200\ MPa$, $\sigma_y \approx 400\ MPa$

True Stress and Strain

Nominal stress and strain are convenient for material testing, but do not take account of changes in geometry. In some circumstances it is preferable to account for geometry correctly, and to use *true stress* and *true strain*:

True Stress

For uniaxial load, *true stress* is: $\sigma_t = \dfrac{F}{A}$ and *nominal stress* is: $\sigma_n = \dfrac{F}{A_o}$

For constant volume (i.e. for plastic deformation): $A_o\,\ell_o = A\,\ell$

Hence: $\sigma_t = \sigma_n\,(A_o/A) = \sigma_n\,(\ell/\ell_o)$

Nominal strain: $\varepsilon_n = \dfrac{\ell - \ell_o}{\ell_o} = \dfrac{\ell}{\ell_o} - 1$, *i.e.* $\dfrac{\ell}{\ell_o} = 1 + \varepsilon_n$

Hence: $\sigma_t = \sigma_n\,(1 + \varepsilon_n)$ i.e. *true stress > nominal stress* (for $\varepsilon_n > 0$).

True stress is easily calculated from nominal stress–strain data, but only *as far as the onset of necking* in tension (deformation no longer uniform) – the continuation of the true stress–strain curve must be found in compression.

True Strain

True strain is more difficult to visualise – each *increment of extension dℓ* is divided by the *current length* (not the initial value), to give an *increment of true strain*: $d\varepsilon = d\ell\,/\,\ell$
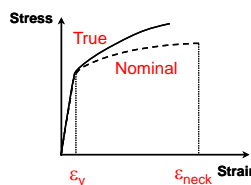
Total true strain is found by integrating between original and final lengths:

$$\varepsilon_t = \int_{\ell_o}^{\ell} \frac{d\ell}{\ell} = \ln\left(\frac{\ell}{\ell_o}\right)$$

Since: $\left(\dfrac{\ell}{\ell_o}\right) = (1 + \varepsilon_n)$, true strain is: $\varepsilon_t = \ln\,(1 + \varepsilon_n)$

i.e. *true strain < nominal strain* (for $\varepsilon_n > 0$), and easily calculated from ε_n.

Comparative *stress-strain curves* (up to maximum nominal stress):

Stress

True

Nominal

ε_y ε_{neck} Strain

When is *nominal $\approx$ true*?

$\varepsilon_t = \ln(1 + \varepsilon_n) = \varepsilon_n$ + higher order terms

$\varepsilon_t \approx \varepsilon_n$ for $\varepsilon_n \ll 1$

e.g. for *yield strain*, ε_y: $\varepsilon_n \approx 0.1\%$

$\varepsilon_t = \ln(1.001) = 0.001$ (3 d.p.'s)

For *necking strain*, ε_{neck}: $\varepsilon_n \approx 20\%$ (e.g. mild steel)

$\varepsilon_t = \ln(1.2) = 0.182$ (3 d.p.'s)   i.e. 18%

Tension vs. Compression

True stress and strain take account of geometry change, so true $\sigma-\varepsilon$ curves for tension and compression are identical.

At yield strain $\varepsilon_n \approx \varepsilon_t$ and $\sigma_n \approx \sigma_t$, hence yield stress will be the *same in tension and compression*.
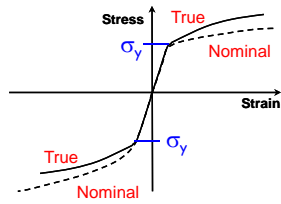
Additivity of true strains

Some manufacturing processes apply sequential plastic strains – e.g. multi-pass rolling. Each deformation can be described by nominal or true strain.

The advantage of *true strain* is that it is *additive*: separate strains can be summed. This is *not so for nominal strain* (see Examples Paper 3).

---

5.3 Data for Strength – Material Property Charts

Strength is defined as the *nominal stress at the elastic limit*.

Strength data are more complex, due to the different ways in which material classes exceed their elastic limit:

- *Brittle materials* (e.g. ceramics) (tension): *fracture* occurs, giving a *tensile strength* (variable, due to variability in internal flaws)

- *Brittle materials* (compression): *crushing* occurs, giving a *compressive strength* (typically 10-15 × higher than tensile strength)

- *Ductile materials* (e.g. metals) (tension or compression): *yield stress* (or proof stress)

Material property charts including Strength

(1) *Strength vs. Density*
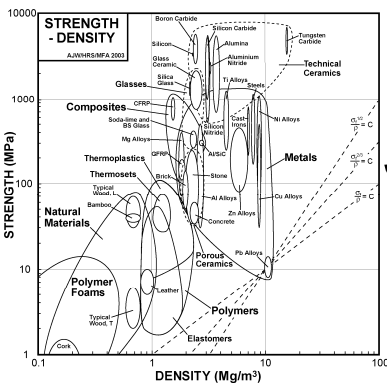
NB: Envelope round *technical ceramics*, *glasses* and *porous ceramics* shown dotted: data for <u>compression only</u>

Performance indices – return to these later.

Notes:

- Strength for solid materials spans over 3 orders of magnitude – natural materials and polymer foams extend the range to much lower strengths;

- Metals, ceramics and composites are the strongest materials, but there is some overlap with polymers;

- Individual classes of metals (e.g. steels, Ti alloys etc) cover much wider ranges of strength than is observed for Young's modulus or density.

---

(2) *Young's Modulus vs. Strength*
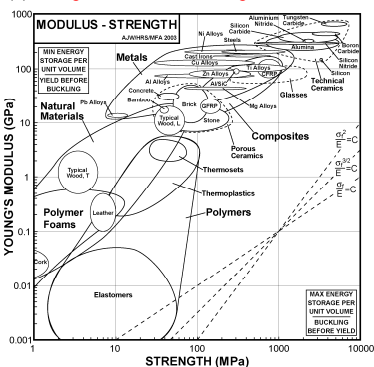
NB. Wide ranges in strength but not in Young's modulus – strength is a property that can be manipulated widely by composition and processing.

Further performance indices – e.g. yield strain, elastic stored energy per unit volume

Materials Databook also includes a property chart for *Fracture Toughness vs. Strength* – discussed in the Easter Term lectures on fracture.